Cooperation and Navigation Model of Self-driving Cars Based on Cellular Automata

This paper mainly discusses the best percentage of the self-driving cars to solve the congestion status in interstate 5, 405, 90 and State Route 520 and how to make the self-driving cars play the greatest role in improving traffic conditions.

Firstly, the data of the problem are used to analyze the traffic conditions according to the two-fluid theory. The flow conservation equation is established to calculate the average equivalent queue length for multi-lane sections. The longest queue length of the four roads are 1.97mi, 1.91mi, 1.35mi, 1.20mi, which are considered as the most congested sections.

Secondly, the model of cellular automata based on safety distance rules is established. Suppose the manual cars take the ACL strategy and the self-driving cars take the PCL strategy by exchanging state information with self-driving car in adjacent lanes so that we can set the boundary conditions of cellular automata. We can find when the proportion of self-driving car is higher than 30.57%, the traffic situation can be improved and when it is 60.34%, the traffic situation is optimal.

Besides, choose the intersection of Interstate 5 and 405 to study the intersection situation. Interstate 5 is considered as the main line and the flow difference value between the road sections is considered as the flow that merges in main line. These parameters are input to the problem 2 in cellular automata for analysis. The equilibria is 30.57%, and the tipping point is 60.34%. Therefore, it can be found that the weaving area has little effect on the critical value and the extreme value.

Last, we take dedicated lane into consideration. Assume those two kinds of cars only travel on their lanes. In the dedicated lane model, the proportion of the self-driving car is considered as 100%, and the safe distance is 0. According to the calculation result, it is reasonable to set one dedicated roads on two-lane sections, two dedicated roads on three-lane sections, three dedicated roads on four-lane sections, three dedicated roads on the five-lane section which can increase the average speed by 3.67% 4.84%, 13.48%, 13.26%.

**Key words:** NaSch Cellular Automaton Driving Model, Lane Change Maneuvers, Politeness Index, Two-fluid theory, Safe distance
1.1 Problem background

Due to the number of lanes, traffic capacity in many regions of the United States is limited. In the Greater Seattle area, drivers are experiencing long delays during peak hours due to traffic volumes exceeding the design capacity of the road network. This is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520, the roads of particular interest for this problem.

Self-driving, cooperative cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads. They can cooperate with each other to have shorter car-following distances, co-lane change and safe lane changing strategies. Moreover, they can interact with the manual cars. Therefore, it is imperative to find an optimum proportion to optimize the traffic efficiency.

1.2 Analysis of the Problem

In order to solve the problem of cooperation and navigation between self-driving and manual driving cars, we will do the following work in turn.

Firstly, we use the data of Interstate 5, 405, 90 and State Route 520 to analyze road traffic conditions. According to the two-fluid theory, the actual running status of the traffic flow is converted into the two-flow status combined with the optimal flow and the blocking flow. The flow conservation equation is established by using the flow of the entrance and exit of each road, and the average density of the traffic flow queuing phenomenon is obtained at this traffic level And then we build the average equivalent queue length model of multi-lane sections, and select the longest queuing sections of four roads, which will be considered as the most congested sections.

Secondly, a model of cellular automata based on safety distance rules is established for the most congested sections. The vehicle is regarded as a regular, solid and limited state (velocity) object. The flow and length of the road sections, the number of lanes and the proportion of self-driving cars are set as the input parameters, and the road is modeled as a one-way open road. Assuming that the vehicle is divided into self-driving cars and manually driven cars according to different safety distances. Manned vehicles take Aggressive Change Lane strategy to achieve their independent expected speeds. In the process of iteration, we suppose the self-driving cars take the Polite Change Lane strategy by exchanging state information with self-driving car in adjacent lanes. According to the two rules, the boundary conditions of cellular automata are set, change the proportion of self-driving cars and output the Time-Space diagram and the average speed, analyze the equilibria and the tipping point.

Thirdly, as the most congested sections are near the intersection of the roads, choose the intersection of Interstate 5 and 405 to study. Consider Interstate 5 as the mainline and the flow difference between the inward and outward sections of Interstate 405 as the input of the on-ramp traffic flow. The right-most lane flow of the weaving area is the sum of the main flow and the incoming flow, and then input into the cellular automata of question 2 to analyze.
Last but not least, in order to minimize the interaction between manual driving cars and self-driving cars, it is considered that there are only self-driving cars on dedicated lanes and other cars only driving on common lanes, setting lane separator to prevent mixed traffic flow. The road segment is considered as two lane groups of common lanes and dedicated lanes, and they are modeled separately. In the dedicated lane model, the self-driving ratio is 100%, the safety distance is 0. The density is calculated using $D_s = 0.6 \times N/n_s$. By changing the input parameters, the average speed of each lane is obtained, and the optimal lane combination is obtained by analyzing the common lane in the same way.

2 Assumptions

To simplify the problem, we make the following assumptions.

- The sections given by the subject are all straight lines, and the vehicle operation is not restricted by the road geometry.
- There is no entrance in the middle of the link, as the length of the link length is short, the inlet flow in startMilepost equals to the outlet flow in endMilepost.
- The length of a single cell is 5m, and the traffic flow only consider the standard small passenger cars, converting other types of vehicles into standard car by conversion factor.
- Suppose the highway is a one-way open road, and its speed limits are equal everywhere.
- Assume that each vehicle has its own pre-set independent expected speed and will not accelerate anymore once the vehicle speed reaches the expected speed. It is assumed that the expected speed of all vehicles is uniformly distributed
- The status of each vehicle is only relevant to the state of the vehicle near it

3 Symbol Definition

3.1 Symbol for model I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0$</td>
<td>The number of vehicles between the upstream and downstream sections at the initial time</td>
</tr>
<tr>
<td>$N_U(t)$</td>
<td>The cumulative number of vehicles passing through the upstream section at time $t$</td>
</tr>
<tr>
<td>$N_D(t)$</td>
<td>The cumulative number of vehicles passing through the downstream section at time $t$</td>
</tr>
<tr>
<td>$\Delta N(t)$</td>
<td>The number of vehicles between the upstream and downstream sections at time $t$</td>
</tr>
<tr>
<td>$L_D(t)$</td>
<td>The equivalent queue length between the upstream and downstream sections at time $t$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>L</td>
<td>The distance between the upstream and downstream sections at time t</td>
</tr>
<tr>
<td>$k_m$</td>
<td>The best density of traffic flow between upstream and downstream sections</td>
</tr>
<tr>
<td>$k_j$</td>
<td>The blocking density of traffic flow between the upstream and downstream sections</td>
</tr>
<tr>
<td>$\overline{L_D}(t)$</td>
<td>the average equivalent queue length between the upstream and downstream sections of the multi-lane section at time t</td>
</tr>
<tr>
<td>$N_U(i,t)$</td>
<td>The cumulative number of vehicles in the upstream section of the i-th lane at time t</td>
</tr>
<tr>
<td>$N_D(i,t)$</td>
<td>The cumulative number of vehicles in the downstream section of the i-th lane at time t</td>
</tr>
<tr>
<td>M</td>
<td>Number of lanes</td>
</tr>
<tr>
<td>$\overline{k}_j$</td>
<td>Average blocking density of single lane</td>
</tr>
<tr>
<td>$\overline{k}_m$</td>
<td>Average best density of single lane</td>
</tr>
<tr>
<td>$\overline{k}(t)$</td>
<td>The average single-lane traffic density between upstream and downstream sections at time t</td>
</tr>
<tr>
<td>PHT</td>
<td>Peak hour traffic flow</td>
</tr>
<tr>
<td>ADT</td>
<td>Average Daily Traffic Volume</td>
</tr>
</tbody>
</table>

3.2 Symbol for model II—IV

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_i$</td>
<td>The current speed of the i-th vehicle</td>
</tr>
<tr>
<td>$t_v i$</td>
<td>The i-th vehicle's target speed</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Lane number</td>
</tr>
<tr>
<td>$t_l i$</td>
<td>The number of the i-th vehicle's target lane</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Cell number</td>
</tr>
<tr>
<td>$(l_i, x_i)$</td>
<td>The location of the i-th car</td>
</tr>
<tr>
<td>$L_i$</td>
<td>The length of the i-th car</td>
</tr>
<tr>
<td>gap$_{head}(l, x)$</td>
<td>The number of consecutive free cells before the cell $(l, x)$</td>
</tr>
<tr>
<td>gap$_{back}(l, x)$</td>
<td>The number of consecutive free cells after the cell $(l, x)$</td>
</tr>
<tr>
<td>$v_n(t)$</td>
<td>Speed of the n-th car at time t</td>
</tr>
<tr>
<td>$b_n(t)$</td>
<td>Condition of the n-th car at time t</td>
</tr>
<tr>
<td>$p_{b}, p_0, p_d$</td>
<td>Probability parameter</td>
</tr>
<tr>
<td>$d_n(t)$</td>
<td>The distance between the n-th and the (n-1)-th car at time t</td>
</tr>
<tr>
<td>$n_c, n_s$</td>
<td>the number of common lanes, dedicated lanes</td>
</tr>
<tr>
<td>$D_c, D_s$</td>
<td>the traffic density of common lanes, dedicated lane</td>
</tr>
</tbody>
</table>
### 4 Equivalent Queue Length Model for Congested Traffic Stream

#### 4.1 Equivalent queue length model of single - lane road section

The data presented in this question includes the startMilepost and endMilepost for some of the sections in Interstates 5, 90, 405 and State Route 520, and the average daily traffic volume.

According to the conservation of flow principle, we can see

\[ N_0 + N_U(t) = N_D(t) + \Delta N(t) \]  \hspace{1cm} (1)

The traffic flow is divided into blocking traffic flow and optimal traffic flow. When the traffic flow condition is optimal, the density reaches the optimal density \( k_m \). When the traffic flow appears stable queuing phenomenon, the blocking flow length is the vehicle equivalent queue length \( L_D(t) \), when the density reaches the blocking density \( k_j \)

\[ \Delta N(t) = k_j L_D(t) + k_m [L - L_D(t)] \]  \hspace{1cm} (2)

By solving equations (1) and (2)

\[ L_D(t) = \frac{N_0 + N_U(t) - N_D(t) - k_m L}{k_j - k_m} \]  \hspace{1cm} (3)

In a single road, the vehicle queue length cannot be less than 0, and cannot exceed the length of the road, that is \( 0 \leq L_D(t) \leq L \).

#### 4.2 The average equivalent queue length model of multi-lane sections

The roads mentioned before are multi-lane sections, there is overtaking phenomenon. To simplify the model, all lanes are combined as a lane group, which become a single-lane road without intermediate entrances. Although the equivalent queuing length of each lane is not the same, an average queue length can be calculated to describe the equivalent queue length over a multi-lane segment as a whole. As the ADT data in the title is not divided into directions, this paper combines the two-way lane to calculate the overall average queue length.

The average queue length of i lanes is deduced as follows
The above analysis shows that the blocking density $k_j$ and the optimum density $k_m$ need to be calibrated according to the actual data. According to HCM2000 multi-lane highway service level rating definition, when the road is blocked, we take the maximum range of the service level D-class density interval, which is 256pcu / (mile * ln), as the blocking density.

The most suitable model for describing the congested traffic flow is the Greenberg model, and the best density is $k_j/e$, which is 52pcu / (mile * ln).

8% of the daily traffic volume occurs during peak travel hours on average. The data for the i-th road segment is processed according to the following equation.

$$L_D(t) = \frac{N_0 + \sum_{i=1}^{M} N_U(i,t) - \sum_{i=1}^{M} N_D(i,t) - k_m L M}{M(k_j - k_m)}$$

(4)

4.3 Queue length calculation

Find the most congested road sections and queue lengths for each road as shown in the table below.

Table 4.1 The queue length of the most congested sections of each road

<table>
<thead>
<tr>
<th>Route_ID</th>
<th>serial number</th>
<th>startMilepost</th>
<th>endMilepost</th>
<th>length/mile</th>
<th>queue length/mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>49</td>
<td>138.04</td>
<td>141.64</td>
<td>3.6</td>
<td>1.97</td>
</tr>
<tr>
<td>405</td>
<td>207</td>
<td>27.4</td>
<td>29.88</td>
<td>2.48</td>
<td>1.91</td>
</tr>
<tr>
<td>520</td>
<td>219</td>
<td>6.93</td>
<td>9.6</td>
<td>2.67</td>
<td>1.35</td>
</tr>
<tr>
<td>90</td>
<td>149</td>
<td>10.15</td>
<td>11.64</td>
<td>1.49</td>
<td>1.20</td>
</tr>
</tbody>
</table>

As the equivalent queue length depends on the number of vehicles on the road, the more the number of vehicles, the greater the average traffic density. From the characteristics of traffic flow, traffic density is one of the important parameters to measure traffic congestion. Therefore, equivalent queue length can also be used as an indicator of the degree of congestion.
From the above figure, in the section of the unit length, with the increase of the difference of the flow of the inlet and outlet section, the actual traffic volume of the section is larger than its capacity, so its queue length increases.

The location of the most congested roads is labeled in the figure below. They are all in the vicinity of the intersection of road segments. The following parts will focus on analysis of these sections.

![Figure 4.2 The most congested road sections of the road location](image)

### 5 Mixed Traffic Flow Model Based on NaSch Cellular Automata

#### 5.1 Establishment of Manual Driving Model

Using the cellular automata model, the corresponding section of the highway is modeled as M column cells, so that each column corresponds to a lane on the expressway, and the cells on each column start from 1 in the vehicle running direction. The contents are shown in Figure 5.1.

![Figure 5.1 Schematic of the rules](image)
(1) lattice road
The number of pairs \((l_i, x_i)\) represents the location of the i-th vehicle, and each "vehicle" occupies a cellular grid as follows:

\[
\text{road}(l_i, x_i) = \begin{cases} 
  v & \text{(If the cell has a car and the speed is } v) \\
  0 & \text{(no car)} 
\end{cases}
\]

The model is discrete in time. During iteration, the position of each vehicle is updated according to the following procedure. When the iteration is complete, the time advances one unit.

(2) Velocity
Each car has a speed, and this speed has a lower limit and upper limit

\[1 \leq \text{road}(l_i, x_i) \leq 5\]

1 represents 5m / s, and 20 represents 25m / s. Here, taking 1 rather than 0 is because it can distinguish between car-free grid points and car grid points.

Each car occupies a step (1s), and if there is no other car in front of the \(v\) grid, it will walk \(v\) lattice, that is:

\[
\text{road}(l_i, x_i - v) = \text{road}(l_i, x_i); \ \text{road}(l_i, x_i) = 0
\]

(3) Rules for transverse lane changing
Drivers sometimes choose to divert over other vehicles or enter a more unimpeded area. Once a vehicle decides to change lanes, it immediately moves laterally to adjacent cells on adjacent lanes, provided that the target cell is also empty, otherwise lane change decisions are abandoned. As long as there is one of the four conditions of \(\text{road}(i, j - k) > 0 (k = 1,2,3,4)\), the driver may choose to divert

\[
\ll \text{road}(l_{i1}, x_{i1}), \text{road}(l_{i2}, x_{i2}) \gg = p_{i1i2}
\]

\(p_{i1i2}\), which is the probability of diversion from \(i_1\) to \(i_2\) (assuming that \(p_{i, i}\) is the probability of invariant channel), it is obvious that the following relations are satisfied:

\[
\sum_{j=1}^{M} p_{i,j} = 1, p_{i,j} \geq 0
\]

This formula expresses the conservation of the vehicle. The following equation shows that it is not possible to skip Lane 2 and change lanes directly between 1 and 3

\[p_{1,3} = p_{3,1} = 0\]

When the following conditions are met, the vehicle performs a lane change action:

- The front of the car makes the car cannot continue to accelerate or be forced to slow down the brake

\[
\text{gap}_{\text{head}}(l_i, x_i) \leq v_i
\]

- Adjacent lanes are better than the lane, such as no vehicles or vehicles in front of the adjacent lane farther than the current vehicle

\[
\text{gap}_{\text{head}}(l_{i1}, x_{i1}) \leq \text{gap}_{\text{head}}(l_{i2}, x_{i2})
\]

- There is no vehicle on the adjacent lane to block the lane change of the own vehicle
\[ \text{gap}_{\text{back}}(t_i, x_i) \geq L_i \]

When the above conditions are satisfied at the same time, the position of the vehicle is updated as follows

\[ (l_i, x_i) \rightarrow (t_l, x_l) \]

Fig. 5.2 An example of lateral lane change decision process

(4) The longitudinal advance rule

When all the vehicles have completed the decision and execution of the lane change, they will proceed at the same time. The car will be accelerated if there is no car in the front \( v \) lattice. The car will be added to the speed \( v +1 \) after move \( v \) lattice. However, due to highway speed limit, we stipulate that the maximum \( v \) can only reach 5. The model assigns a separate desired speed to each vehicle by the parameter \( v_{\text{exp}} \)

Let

\[ t_v = \min(v_{i+1}, v_{\text{exp}}); \quad t_v = \min(t_v, \text{gap}_{\text{head}}(l_i, x_i)) \]

If \( t_v < v_i \), then \( t_v = \max(t_v - 1, 0) \), \( p = p_{\text{overbrake}} \).

When the above conditions are satisfied at the same time, the position of the vehicle is updated as follows

\[ v_i \rightarrow t_v; \quad (l_i, x_i) \rightarrow (l_i, x_i + v_i) \]

5.2 Full manual driving simulation and numerical analysis

Do simulation for marked sections of interstate 5, 90, 405 and state road 520 in (1, 3). Taking Interstate 5 and Route 49 as an example, we use the periodic boundary conditions, set \( N \) cars distributed evenly on the road at the initial state, and let the vehicle's initial speed of 0km/h, the total traffic density on the road \( \rho \), \( \rho = N / L \).

<table>
<thead>
<tr>
<th>Route_ID</th>
<th>serial number</th>
<th>startMilepost</th>
<th>endMilepost</th>
<th>length/mile</th>
<th>Average daily traffic counts Year_2015</th>
<th>queue length/mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>49</td>
<td>138.04</td>
<td>141.64</td>
<td>3.6</td>
<td>190000</td>
<td>1.97</td>
</tr>
</tbody>
</table>

Simulate until the peak hour flow all go through the section. Record all speeds in the last 1000 time steps, find the average speed of the vehicle in each step, and then average the resulting velocity again to obtain the average speed of one run.
Table 5.2 The list of simulation parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of lanes</td>
<td>4</td>
<td>SPDmax</td>
<td>5 cells/step</td>
</tr>
<tr>
<td>Lane length</td>
<td>3.6 mile = 1150 cells</td>
<td>SPDmax</td>
<td>1 cell/step</td>
</tr>
<tr>
<td>Number of vehicles</td>
<td>15200</td>
<td>poverbrake</td>
<td>0.5</td>
</tr>
<tr>
<td>Expected speed</td>
<td>[SPDlow, SPDmax] (Evenly distributed)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The above parameters and the headway $d_{safe}$ (set as 5m) are imported into the cellular automata model built in MATLAB to simulate the traffic flow.

Figure 5.3 Traffic Flow Diagram of the Most Congested Section of Interstate 5

Fig 5.4 space-time chart of the most congested sections at interstate 5

The average relative speed is 8.93. The trend of the space-time diagram should be from the bottom left to the top right. However, the crowded road and frequent stop of the vehicles make the space-time map showing a reverse trend from the upper left to lower right. So we can determine the road in a more serious congestion condition.

The other three sections of the analysis are similar, so will not be discussed here.

5.3 Establishment of Mixed - Flow Model of Manual - Automatic Driving Based on Safety Distance

The safe driving distance of the two models is shown below:

Fig 5.3 Diagram of safety distance between manual driving and self-driving
The simulation process of each traffic flow consists of several iterations. A round of iterations represents a unit time, with iteration consisting of a lateral lane change action and a longitudinal advance motion.

(1) Horizontal lane changing rules

The corresponding lane change model is executed for all cars. A lane change decision is made by self-driving cars when the following condition is satisfied.

- The front car makes the rear of the car cannot continue to accelerate or be forced to slow down the brake
  \[ \text{gap}_{\text{head}}(l_i, x_i) \leq v_i \]

- Adjacent lanes are better than the lane. That is to say, there is no vehicle in front of the adjacent lane, or the vehicle is farther away than the current vehicle.
  \[ \text{gap}_{\text{head}}(t_{l_i}, x_i) \leq \text{gap}_{\text{head}}(l_i, x_i) \]

- There is no vehicle on the adjacent lane to obstruct the lane change behavior of the own vehicle
  \[ \text{gap}_{\text{back}}(t_{l_i}, x_i) \geq L_i \]

The co-lane change is to be completed by two self-driving cars in the adjacent lane. If the adjacent lane target cell has been occupied by a vehicle, the vehicle satisfying the conditions 1 and 2 sends a lane request to the other party, and the partner decides whether to accept the request according to the current position after receiving the lane change request. The following assumptions are made for cooperative lane changes:

a) The partner is traveling at the current speed and has been able to exceed the requester after the next iteration. Or even if it decelerates to a standstill, the requester is still unable to surpass it at the next iteration, ignoring the request and proceeding as originally planned.

b) When the partner cannot exceed the requester in a short time, it will accept the request to slow down until sufficient space is reserved for the preceding vehicle to complete the lane change. Otherwise, the partner slows down to a specific speed, allowing the requester to override it before the next iteration to complete the lane change.

That is to say, if \( \text{gap}_{\text{back}}(t_{l_i}, x_i) \geq L_i \), and the co-lane change function is enabled and the vehicle blocking the target lane is also a self-driving car, the vehicle applies for co-lane change and terminates the lane change decision

- The random value \( \text{rand}() \) which represents the willingness of lane change is less than the random factor of lane change decision
  \[ \text{rand}() < p_{cl} \]

In the manual driving model, an Aggressive Change Lane (ACL) strategy is adopted, which does not take into account the traffic conditions on the rear of the target lane. In order to change this situation, this article stipulates that the self-driving car adopts the policy of "politeness change Lane (PCL)", which means the probability of vehicle lane change is determined by the degree of influence of the lane changing and the degree of courtesy.

The probability of change lane \( p_{cl} \) is affected by the speed of the vehicle, the
speed and distance of the rear vehicle. Let the degree of the rear-road vehicle be $\alpha_j$,

$$\alpha_j = \frac{\Delta v_j}{v_{exp_j}} \in [0,1]$$

When $\alpha = 0$, it means that the vehicle is completely unaffected by the lane changing behavior. When $\alpha = 1$, it means that the vehicle speed is reduced from the desired vehicle speed to zero.

The polity coefficient $pol \in [0,1]$. The lane change probability of the i-th vehicle is determined by $pol$, and the degree of influence $\alpha_j$ of the rear j on the target lane.

$$p_{cl,i} = f(\alpha_j, pol)$$

$$p_{cl,i} = \begin{cases} 1, & \text{if } \alpha_j = 0 \\ 0, & \text{otherwise if } pol = 1 \\ \max \left( -\frac{pol}{1-pol}, \alpha_j + 1,0 \right), & \text{otherwise} \end{cases}$$

When the above conditions are satisfied at the same time, the position of the vehicle is updated as follows:

$$(l_i, x_i) \rightarrow (tl_i, x_i)$$

(2) Vertical advance rules

Due to the cooperative role between self-driving cars, the running state of the rear vehicle is affected by the front vehicle. Starting from the foremost end of the cell group, the sequential forward movement is performed sequentially for each networked self-driving car.

Let

$$tv_i = \min(v_{i+1}, v_{exp_i}); \ tv_i = \min(tv_i, gap_{head}(l_i, x_i))$$

If the co-lane request flag is not empty and the requester is the j-th vehicle, the flag is cleared.

When the above conditions are satisfied at the same time, the position of the vehicle is updated as follows:

$$v_i \rightarrow tv_i; \ (l_i, x_i) \rightarrow (l_i, x_i + v_i)$$

5.4 Simulation and Numerical Analysis of Mixed Flow

Do simulation for marked interstate 5, 90, 405 and state road 520 in (1, 3). The data of the most congested road sections are put into the cellular automata which has been added into the rules of autopilot vehicles for simulation and numerical analysis.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lane change allowed</td>
<td>true/false</td>
</tr>
<tr>
<td>Co-lane change allowed</td>
<td>true/false</td>
</tr>
<tr>
<td>Polite Coefficient</td>
<td>pol</td>
</tr>
<tr>
<td>Proportion of self-driving car</td>
<td>(\theta)</td>
</tr>
<tr>
<td>expected departure interval for a single lane</td>
<td>(\lambda)(Poisson distribution)</td>
</tr>
</tbody>
</table>
The above parameters and the headway $d_{safe}$ are imported into the cellular automata model built in MATLAB to simulate the traffic flow. From 0-1 to change the ratio of the self-driving car $\theta$, the relative velocity is obtained as shown in the following table:

Table 5.5 The relative velocity of different percentage of self-driving cars

<table>
<thead>
<tr>
<th>percentage of self-driving car</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative velocity</td>
<td>8.93</td>
<td>8.65</td>
<td>8.86</td>
<td>8.91</td>
<td>8.97</td>
<td>9.01</td>
</tr>
<tr>
<td>percentage of self-driving car</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Analyze the proportion of self-driving cars on the impact of speed, resulting in the relative speed trend, as shown below:
From the relative velocity trend graph, we find that when the proportion of the autopilot is 31.15% or less, the overall traffic efficiency is not improved due to the interaction with the manual vehicle.

When the proportion of automatic driving is higher than 59.45%, the vehicle traffic efficiency has increased significantly. When the ratio grow, the trend of relative speed is flat, which shows that when the proportion of self-driving cars reach 60%, the queuing will be dissipated basically and the congestion will be alleviated effectively. What’s more, traffic flow can be free flow and it will not restricted by front and rear vehicle. There is no need to increase the ratio to improve traffic.

Fig. 5.4 relative velocity of each road

Fig. 5.5 relative velocity in different percentage of self-driving car

From the above analysis available, it’s shown that the number of self-driving cars which accounted for more than 30% of road traffic flow can improve traffic conditions. When the proportion rose to 60%, it’s the critical point that can solve the congestion. The proportion can basically solve the congestion situation.
Fig 5.6 Space-time chart of 60% self-driving cars in the most congested sections of I5#

From the figure above we can see that the trend of the space-time chart is basically presented from top left to bottom right when self-driving cars counting for 60%. And the spatiotemporal point is relatively sparse, you can determine the road congestion situation greatly alleviated

6 Establishment of Ramp Traffic Flow Model

6.1 Model Building

Ramp Traffic Flow Model is same as traffic flow model in Construction of Cellular Automata, Acceleration / Deceleration Rule, Horizontal Shift to Rule and Longitudinal Advance Rule. The difference between the ramp traffic model and the road model is that the interweaving of the ramp model is more frequent and complex. The cellular automata model under ramp condition can be obtained by simply adding the random slowdown parameter in the model construction. The process of determining the random moderation parameter \( p_n(t + 1) \) is as follows:

\[
p_n(t + 1) = p(v_n(t), b_{n+1}(t), t_{h,n}, t_{s,n})
\]

\[
p(v_n(t), b_{n+1}(t), t_{h,n}, t_{s,n}) = \begin{cases} 
 p_b (b_{n+1}(t) = 1, t_{h,n} < t_{s,n}) \\
 p_0 (v_n(t) = 0, t_{st,n} \geq t_c) \\
 p_d ( \text{else}) 
\end{cases}
\]

\[
t_{h,n} = \frac{d_n(t)}{v_n(t)}, t_{s,n} = \min(v_n(t), h).
\]

6.2 Simulation and Numerical Analysis of weaving area

In the analysis of road traffic flow model, it is found that the four roads are the same. Therefore, take the weaving area of interstate 5and interstate 405 as an example. Using periodic boundary conditions, there are N cars on the road in the initial state and the initial speed of the vehicle is 0 m/s. The traffic density of the right-most road section is different from that of the road section because of the ramp-in traffic. In addition to the original vehicle, the rightmost lane also includes the number of vehicles imported from the ramp. Therefore, the right-most lane density is \( \rho = \frac{N_r + N}{L} \), and \( N_r \) is the number of ramp Imported cars. The traffic volume in the weaving
area and the ramp-in traffic volume in peak hours are shown in Table 6.1.

Table 6.1 Confluence area basic data of interstate 5 and 405

<table>
<thead>
<tr>
<th>Route_ID</th>
<th>traffic counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>17920</td>
</tr>
<tr>
<td>ramp</td>
<td>2320</td>
</tr>
<tr>
<td>405</td>
<td>6000</td>
</tr>
</tbody>
</table>

Simulation of the evolution of each run until peak hour traffic all go through the road. Record all speeds in the last 1000 time steps and the average speed of the vehicle in each time step is obtained. Then the resulting velocity values are averaged to get the average speed of one run. The model input parameters are shown in Table 6.2.

Table 6.2 The list of simulation parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of lanes</td>
<td>5</td>
<td>SPDmax</td>
<td>5 cells/step</td>
</tr>
<tr>
<td>Lane length</td>
<td>2.46mi=786cella</td>
<td>SPDmax</td>
<td>1 cells/step</td>
</tr>
<tr>
<td>Number of vehicles on Interstate 5</td>
<td>15200</td>
<td>poverbrake</td>
<td>0.5</td>
</tr>
<tr>
<td>Number of cars on the ramps</td>
<td>2320</td>
<td>Random moderation parameter</td>
<td>0.7</td>
</tr>
<tr>
<td>Percentage of self-driving car</td>
<td>0-1</td>
<td>Expected speed ([SPDlow,SPDmax])</td>
<td>(Evenly distributed)</td>
</tr>
</tbody>
</table>

The above parameters are substituted into the cellular automata model built in MATLAB and the proportion of the self-driving car is increased gradually. The following traffic flow diagram is obtained.
Fig. 6.1 Space-time chart of weaving area in peak hours

Figure 3.1 shows the Space-time chart when the proportion of the self-driving car is 20%, 40%, 70% and 90%. From the time-space map, it can be found that the time of congestion of the traffic flow decreases after increasing the proportion of the self-driving car and the traffic flow can run at a steady speed for a long time.

Relative velocity change line chart shown in Figure 3.2 can be obtained through the relative velocity output by the MATLAB program.

![Relative velocity change line chart](image)

From this figure, it can be found that when the proportion of the self-driving car is 60%, the relative speed reached a peak. The impact on speed is not obvious when ratio continues to rise. Therefore, when the proportion of self-driving car in about 60%, costs are low and significant results can be achieved. At the same time, if the proportion of the self-driving car is low, the relative speed will decrease until the proportion of the self-driving car reaches 30% which indicates that there is a strong interaction between the manual car and the self-driving car. Until the proportion of the self-driving car reaches a certain value, the benefits of auto production is greater than the impact of interaction.

7. The analytical model of dedicated lanes for self-driving cars

7.1 Model Building

In the analysis of model 2 and model 3, it is found that the traffic efficiency is the highest when the proportion of the self-driving cars is 60%. Therefore, the situation when the proportion of self-driving cars is 60% is taken as an example to explore the optimal number of dedicated lanes. There is interaction between the self-driving cars and non-automatic cars and the interactive effect is obvious when the proportion of the self-driving cars is small. Therefore, to maximize the traffic efficiency of the road section, it is assumed that the self-driving cars only run on the dedicated lanes and the manual cars only run on the common lanes.

Under this assumption, the cellular automata model is established for the
dedicated lanes and the common lane respectively. The average relative velocity of the two kinds of lane under the different assumption of the number of lanes is calculated according to the car ratio (0.6:0.4).

(1) Parameter calculation of dedicated lanes

\[ n = n_s \]
\[ D_s = 0.6 \times N/n_s \]
\[ V_s = \frac{\sum_{i=1}^{N_s} V_i}{N_s} \]
\[ \bar{V}_s = \frac{\sum_{i}^{500} V_{si}}{500} \]
\[ N_s < N \]

(2) Parameter calculation of common lanes

\[ n = n_c \]
\[ D_c = 0.4 \times N/n_c \]
\[ V_c = \frac{\sum_{i=1}^{N_c} V_i}{N_c} \]
\[ \bar{V}_c = \frac{\sum_{i}^{500} V_{ci}}{500} \]
\[ N_c = N - N_s \]

The simulation parameters are shown in Table 7.2.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of lanes</td>
<td>( n_c )</td>
<td>SPDmax</td>
<td>5 cells/step</td>
</tr>
<tr>
<td>Lane length</td>
<td>1000 cells</td>
<td>SPDmax</td>
<td>1 cell1/step</td>
</tr>
<tr>
<td>Vehicle density</td>
<td>( D_c )</td>
<td>poverbrake</td>
<td>0.5</td>
</tr>
<tr>
<td>safe distance</td>
<td>1 cell</td>
<td>steps</td>
<td>500</td>
</tr>
<tr>
<td>Expected speed</td>
<td>[SPDlow,SPDmax]</td>
<td>(Evenly distributed)</td>
<td></td>
</tr>
</tbody>
</table>

(3) Calculation of the overall average relative velocity

\[ \bar{V} = 0.6 \times \bar{V}_s + 0.4 \times \bar{V}_c \]

Where \( \bar{V} \) is the overall average speed of the road section.

### 7.2 Simulation and Numerical Analysis of Dedicated Lane Model

Taking the parameters in part 4.1 into the cellular automaton model, we can get the space-time chart shown in Fig 7.1.
Fig 7.1 Space-time chart when set dedicated sections

Compared with the Space-Time diagram before setting the dedicated lanes, it is found that there is no congestion in the road section, and the time-space distribution is uniform. Besides, the speed of each car is evenly advanced, indicating that the cars can advance at a steady speed. We can conclude that setting up dedicated lanes can improve the efficiency of road sections.

In order to analyze the specific effect of road traffic efficiency improvement and get the number of dedicated lanes intersections that have different total lanes, use the formula of $V_c, \overline{V}_c, V_S, \overline{V}$ in 4.1 to calculate. The result is shown in Table 7.3.

Table 7.3 The average speed of different number of dedicated lanes

<table>
<thead>
<tr>
<th>accommodation lane</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative velocity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>9.10</td>
<td>11.04</td>
<td>14.23</td>
</tr>
<tr>
<td>1</td>
<td>9.46</td>
<td>12.44</td>
<td>14.81</td>
</tr>
<tr>
<td>2</td>
<td>9.54</td>
<td>12.41</td>
<td>15.57</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>12.53</td>
<td>16.12</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>15.01</td>
</tr>
<tr>
<td>percentage of increase</td>
<td>4.84%</td>
<td>13.48%</td>
<td>13.26%</td>
</tr>
</tbody>
</table>

On two lane sections, the setting of dedicated lanes has improved the efficiency of road sections. Therefore, set one dedicated lane; It can be found in table 4.3 that it is reasonable to set two dedicated roads on three-lane sections, three dedicated roads on four-lane sections, three dedicated roads on the five-lane section which can increase the average speed by 4.84%, 13.48%, 13.26%.
8 Conclusion

This paper focuses on the future development of urban roads in Washington State, and the characteristics of self-driving cars being researched by the major corporations in the future. This paper studies the traffic conditions of autonomous vehicles and manned vehicles.

In theory, we consider that autopilot vehicles usually travel within speed limits. Faster reaction time allow them to act more closely together and act more randomly than humans, which in some cases tends to overreact. At the tactical level, the self-driving car will adopt a more rational behavior in terms of selecting the best route, taking into account obstacles and traffic density, because of the interaction between the vehicles, so that they can quickly change lanes or driving behavior.

The simulation results are shown below: On the highway, when the proportion of self-driving car is 60%, the overall traffic condition is better. But the number of self-driving car is not as good as possible, there is a minimum ratio limit, that is, when the proportion of self-driving car is less than 30%, the interaction between the autopilot and the manned vehicle will have a negative effect. When the proportion of self-driving car is too low, it will have a negative impact on the smooth flow of traffic system.

Automated vehicles cost too much, almost 10 to 20 times the cost of ordinary cars, but the Washington state economy ranked first in the United States, which has the most powerful economic growth. Taking into account the socio-economic conditions behind the state of Washington, it has the ability to afford the development of self-driving cars. In summary, we recommend that Washington state should increase the proportion of the development of self-driving car to 30% to 60%.

Admittedly, this article has some shortcomings. Based on this topic, there are many aspects that can be further studied in the future, such as increasing the content of traffic information to study the reliance letter of automatic driving vehicle to network communication, so as to further enhance the safety level; or increase the headway, vehicle speed Data to study the safety performance of vehicles, thereby further enhancing the safety capacity.

9. Strength and weakness

9.1 Strengths

a) Easy to understand. A more complex system can be described as some very common rules and there is a certain degree of predictive effect.

b) The specific state of the vehicle can be described, and can be presented in the form of animation.

c) The built-in parameters are able to determine a system pattern. Therefore, by selecting the right parameters, detailed data can be got.

d) Various probabilities are introduced to the model, and the lane change,
acceleration, deceleration and other common behavior are taken into consideration.

e) Based on the NaSch cellular automaton driving model, we propose a self-driving model to characterize the self-driving car. And then build a simulation platform for the mixed traffic flow of the self-driving car and the manual car to simulate a simple motorway driving scenarios. On the model, we propose an adjustable pollin-lane change strategy (PCL). By configuring the courtesy coefficient, the self-driving car can be selected in the aggressive and polite lane changing style. From the detailed simulation, we compare the polite lane changing strategy with the basic lane changing strategy (never-changing NCL vs. aggressive changing lane ACL), and evaluate its impact on the efficiency and safety of traffic flow based on a set of evaluation system.

9.2 Weaknesses

a) There are different types of cars on the actual road. The volume of large trucks and its operating speed have large difference compared with cars. In our model, the PCU is used as a vehicle unit, and the road is divided into small cells, which is equivalent to considering only one type of vehicle. Therefore, in order to better simulate the actual situation, different models should be discussed hierarchically.

b) Drivers in real life will consider many more factors in deciding whether to accelerate or decelerate, such as the distance to the front of the lane to be changed, or whether you need to turn at the next intersection or go straight. In this model, if the conditions permit, the car will continue to accelerate until approaching the speed limit. But in real life, the driver will choose the right speed based on their driving habits. Therefore, this model has disadvantages in terms of speed settings.

c) The lane changing strategy of the self-driving car mainly considers the co-lane changing strategy with the adjacent lane. When the self-driving car is adjacent to the manual car, there is a lack of lane changing strategy due to the large randomness of manual driving cars. The vehicle lane change process is divided into horizontal lane changing and longitudinal advancement. The longitudinal displacement in the course of lateral lane change is not taken into account and is not consistent with the actual situation in the case of large vehicle speed.
10 A Letter for the Governor’s office

Dear governor:

Hello, thank you for taking the time to read my letter. We hope that some of the opinions in this letter will be benefit for your decision-making in urban traffic management.

To improve the capacity of transport services is an important measure to protect the rights of citizens. However, as we know, due to road capacity constraints, road traffic demand during peak hours often exceeds basic capacity, leading to prolonged traffic delays and time lost to citizens. From the 2016 annual traffic index, the Seattle area's overall traffic congestion level of 31%, ranked fourth in the United States, late peak congestion level is ranked second in the United States. According to TomTom statistics, due to traffic congestion, the Seattle area drivers a day increased by 39 minutes travel time, an increase of 148 hours a year travel time. In the state of Washington, this is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520, the roads of particular interest for this problem.

Therefore, we consider that it may be possible to reduce the headway by increasing the interaction of information between vehicles, predicting traffic and road infrastructure information in advance, changing passive traffic to active traffic, or consolidating vehicles with consistent speed into the fleet, increase capacity without expansion of the capacity.

This conjecture is possible. In recent years, Google has tested and developed autonomous driving cars in California, where self-driving cars are able to cooperate with each other to obtain real-time traffic information and information on the status of road facilities, reducing safety distances and increasing road service capacity. Not only that, self-driving cars can also form an autonomous system for coordinated driving, in order to improve the overall driving efficiency. Another example is the Active Cruise Contron with Stop & Go function developed by BMW to keep the vehicle at the preset speed. When detecting the front vehicle, the system will adjust the vehicle speed adaptively and ensure the distance from the front car is always maintained at safe range. From a technical point of view, the development of self-driving is feasible.

In a world where only self-driving cars are on the road, the computer will have complete control over the traffic. But because of the cost and the reality of the restrictions, the road cannot all self-driving cars. At present, to avoid traffic congestion, we need to understand how autopilot and manual driving vehicles will interact, which requires studying the impact of the proportion of self-driving cars on roads to road capacity and traffic jams. In order to explore the optimal proportion and optimal traffic rule for the problem of automatic traffic jam mitigation, we establish a mathematical model to analyze it.

Self-driving cars and ordinary vehicles driving rules are not the same. In this paper, we use the cellular automaton model to simulate the mixed traffic flow, and simulate the characteristics of the autopilot and the driving vehicle, such as
acceleration, deceleration, forward, overtaking, etc. The cellular automaton model can be understood in this way: On a plurality of open roads, a lane is represented as a one-dimensional array of cells, and cells on each column are advanced in the vehicle traveling direction. The different rules are used to make the lane selection and lane change in different vehicles. The running states of the self-driving and the manned vehicle in different proportions are simulated to obtain the spatio-temporal map and the density flow diagram of the vehicle. Based on this, we can calculate the traffic conditions in different states, and then analyze the impact of different reaction time on road capacity, and get the relevant conclusions.

Our conclusion is: On the highway, when the proportion of self-driving car is 60%, the overall traffic condition is better. But the number of self-driving car is not as good as possible, there is a minimum ratio limit, that is, when the proportion of self-driving car is less than 30%, the interaction between the autopilot and the manned vehicle will have a negative effect. When the proportion of self-driving car is too low, it will have a negative impact on the smooth flow of traffic system.

In theory, we consider that autopilot vehicles usually travel within speed limits. Faster reaction time allow them to act more closely together and act more randomly than humans, which in some cases tends to overreact. At the tactical level, the self-driving car will adopt a more rational behavior in terms of selecting the best route, taking into account obstacles and traffic density, because of the interaction between the vehicles, so that they can quickly change lanes or driving behavior.

So a self-driving car might make it easier for a mathematician to work. Randomness is often introduced into the model to include unpredictable human behavior. With less uncertainty, self-driving car systems should be simpler than equivalent human-driven traffic. We can accurately predict how individual vehicles respond to events.

Automated vehicles cost too much, almost 10 to 20 times the cost of ordinary cars, but the Washington state economy ranked first in the United States, which has the most powerful economic growth. Taking into account the socio-economic conditions behind the state of Washington, it has the ability to afford the development of self-driving cars. In summary, we recommend that Washington state should increase the proportion of the development of self-driving car to 30% to 60%.
Reference


