2011邀请赛C评阅要点

cxd

设零件A的标定值为x_0, 若加工精度为k, 则零件参数X是区间$[x_0-kx_0, x_0+kx_0]$上的均匀分布。同样的, 若零件B的标定值为y_0, 若加工精度为k, 则零件参数Y是区间$[y_0-ky_0, y_0+ky_0]$上的均匀分布。

产品参数$Z=f(X,Y)=XY$是$[x_0-kx_0, x_0+kx_0]\times[y_0-ky_0, y_0+ky_0]$上的二维均匀分布。记$S=2kx_0\times 2ky_0$是该分布区域的面积, 则正品率$p_0$、次品率$p_1$和废品率$p_2$各为

$$p_0=\frac{\iint_{|Z-1|\le r_1}\mathrm{d}x\mathrm{d}y}{S},\quad p_1=\frac{\iint_{r_1\le|Z-1|\le r_2}\mathrm{d}x\mathrm{d}y}{S},\quad p_2=\frac{\iint_{|Z-1|\ge r_2}\mathrm{d}x\mathrm{d}y}{S}.$$

易知有$p_0+p_1+p_2=1$. 展开计算, 有

$$p_0=\frac{\int_{y_0-ky_0}^{y_0+ky_0}\int_a^b\mathrm{d}x\mathrm{d}y}{2kx_0\times 2ky_0},\tag{1}$$

其中, $a=\max(x_0-kx_0,(1-r_1)/y)$, $b=\min(x_0+kx_0,(1+r_1)/y)$.

$$p_2=\frac{\int_{y_0-ky_0}^{y_0+ky_0}\int_e^{x_0+kx_0}\mathrm{d}x\mathrm{d}y+\int_{y_0-ky_0}^{y_0+ky_0}\int_{x_0-kx_0}^{f}\mathrm{d}x\mathrm{d}y}{2kx_0\times 2ky_0},\tag{2}$$

其中,

$$e=\max(x_0-kx_0,\min(x_0+kx_0,(1+r_2)/y)),$$
$$f=\min(x_0+kx_0,\max(x_0-kx_0,(1-r_2)/y)).$$

因此, 每一件产品的销售额的期望是$4000p_0+3000p_1+0p_2$; 而同时, 每一件产品的成本为$2000+2c/k$. 所以, 希望平均利润最大即为

$$\max\qquad 4000p_0+3000(1-p_0-p_2)-2000-2c/k,\tag{3}$$

其中, p_0, p_2可由(1)和(2)计算。在这个优化问题中, 决策变量为x_0, y_0和k.

记$F(x_0,y_0,k;c)=4000p_0+3000(1-p_0-p_2)-2000-2c/k$. 则要求利润均值为0等价于求下面函数的根:

$$G(c)=\max_{x_0,y_0,k}F(x_0,y_0,k;c).\tag{4}$$

这是个单变量函数求根问题, 函数值可以计算, 导数不易计算, 可采用割线法。

后面的程序可以求得: 最大平均利润1694.4656, 该值于$x_0=0.973991$, $y_0=1.0266685$, $k=0.006000379$处得到, 此时$p_0=97.2212\%$, $p_1=2.7788\%$, $p_2=0$. 当$c=9.505537166$时, 最大平均利润为0.

```
function c2011
   warning off;
   options = optimset;
   x0 = [ 1 1 0.01]';
   c = 0.833292;
   [x,fv,flag] = fminsearch(@f,x0,options,c);
   x,
   fv = -fv,
   flag
   v = f(x,c,1);
   convg = 0;
   c = 8;
   c2 = 10;
   [x,fv,flag] = fminsearch(@f,x0,options,c);
   [x2,fv2,flag2] = fminsearch(@f,x0,options,c2);
   while ~convg,
       if flag2~=1,
           c2 = c2/2;
           [x2,fv2,flag2] = fminsearch(@f,x0,options,c2);
       else
           cn = c2 - fv2 / ( (fv2-fv) / (c2-c) );
           [xn,fvn,flag2] = fminsearch(@f,x0,options,cn);
           c  = c2;
           fv  = fv2;
           c2 = cn;
           fv2 = fvn;
           if abs(c-c2)<=1e-8, convg=1; end
       end
   end
   c2,
   fv2,
   flag2

function v = f(X,c,opt)
   if nargin<3, opt = 0;      end
   if nargin<2, c = 0.833292; end
   x0 = X(1);
   y0 = X(2);
   k  = X(3);
   r1 = 1/100;
   r2 = 2/100;
   S  = 2*k*x0 * 2*k*y0;
   fp0  = inline( 'min(x0+k*x0,(1+r1)./y) - max(x0-k*x0,(1-r1)./y)', 'y','x0','r1','k' );
   p0   = quad8(fp0,  y0-k*y0, y0+k*y0, 1e-10, 0, x0, r1, k );
   p0   = p0 / S;
   fp2u = inline( '(x0+k*x0) - max(min((1+r2)./y,x0+k*x0),x0-k*x0)', 'y','x0','r2','k' );
   fp2l = inline( 'min(x0+k*x0,max(x0-k*x0,(1-r2)./y)) - (x0-k*x0)', 'y','x0','r2','k' );
   p2u  = quad8(fp2u, y0-k*y0, y0+k*y0, 1e-10, 0, x0, r2, k );
   p2l  = quad8(fp2l, y0-k*y0, y0+k*y0, 1e-10, 0, x0, r2, k );
   p2   = (p2u+p2l) / S;
   p1   = 1 - p0 - p2;
   v    = -( p0*4000 + p1*3000 - 2000 - 2*c/k );
   if opt==1,
       fprintf('\n正品率:  %f\n', p0);
       fprintf('次品率:  %f\n', p1);
       fprintf('废品率:  %f\n', p2);
   end
```