Snowboard Course:
The Optimization of the Halfpipe Shape

Abstract

We determine the shape of the snowboard course, also known as “halfpipe”, to get the production of the maximum “vertical air” and the maximum twist in the air. We import some practical restrictions, like safety and economy, thus the maximum “vertical air” is limited.

We figure out the prototype of a snowboard course referring to the halfpipe standard of FIS (International Ski Federation). The halfpipe prototype is a slanted curved surface, the cross curve of which is two ellipse quarter arcs connected with a bottom segment between them. Its shape is determined by four parameters, namely the angle of inclination, the depth, the width, the length of the bottom segment. Simplifying the snowboarding performance based on the prototype, we take it to be composed of several similar repeated motions, each of which can be separated to two parts: in the U-ramp and in the air. Thus we establish a mathematical model of one single motion and verify its correspondence to the real situation with real kinematic parameters.

Changing the four parameters of the halfpipe prototype, we obtain their influences on the “vertical air”. The result is that “vertical air” increases as the inclination and depth increase, while the width and the length of the bottom plat have little influence on the “vertical air”. With a certain restriction, we can determine the shape of a snowboard course to maximize the production of the “vertical air”.

If the edge of the ramp performs as a short slanted tangent of the ellipse arc, we analyze and find that the stress increases so that the initial angular velocity provided by the torque increases. As a result, we tailor the halfpipe prototype to change its edge into a short and slightly slanted tangent. The angle of the tangent can be calculated with a restriction that the difference of the radius of curvature is within one percent. To keep the center of mass on the ellipse arc before taking off, its length can be estimated.

Finally, taking the consideration of an economic perspective and whether the athletes’ body can withstand the impact when landing, we obtain that it needs to consider the speed limit when pursuing the maximum “vertical air”. Therefore, we evaluate that the maximum “vertical air” is 6m in practical motion, when the inclination reaches 19°.

Keyword: halfpipe, “vertical air”, twist, shape
1. Introduction

The half-pipe is a semi-circular ditch or purpose built ramp (that is usually on a downward slope). Competitors perform tricks while going from one side to the other and while in the air above the sides of the pipe.

In a halfpipe competition, an athlete performs a series of twists and turns on a halfpipe by utilizing its slope. The athlete will be able to complete a routine of the highest difficulties only when he reaches a certain height after the takeoff, so the production of "vertical air" is a critical factor in achieving a high score. The distance of "vertical air" is related to the shape of the halfpipe. Therefore, we need to build mathematical models for the halfpipe in order to optimize its shape so that athletes can achieve the maximum vertical distance needed to perform the most spectacular routines.

Figure 1. Halfpipe course

In the manual book of FIS, we find the standard data of a halfpipe course:[1]

**Halfpipe Definition:**

The halfpipe is a channel constructed in the snow. The bottom of the halfpipe is almost flat and it should be slightly bent with a nice continuation from the transition of the walls. The walls are concave to an almost vertical angle. The halfpipe is orientated directly in the fall line. The riders go from one wall to the other in order to achieve the greatest amplitude and perform the most difficult tricks.
### Chart 1. Technical Data---Oversized Pipe

<table>
<thead>
<tr>
<th>TECHNICAL DATA</th>
<th>MINIMUM</th>
<th>RECOMMENDED</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Inclination</td>
<td>15°</td>
<td>16.5°</td>
<td>18°</td>
</tr>
<tr>
<td>L Length</td>
<td>120 Meter</td>
<td>130 Meter</td>
<td>150 Meter</td>
</tr>
<tr>
<td>W Width</td>
<td>15 Meter</td>
<td>16.5 Meter</td>
<td>19 Meter</td>
</tr>
<tr>
<td>H Inner height walls</td>
<td>4.2 Meter</td>
<td>4.5 Meter</td>
<td>5.0 Meter</td>
</tr>
<tr>
<td>T Transition Radius</td>
<td>5.0 Meter</td>
<td>5.2 Meter</td>
<td>5.8 Meter</td>
</tr>
<tr>
<td>V Vertical</td>
<td>40cm @ 85</td>
<td>50cm @ 85</td>
<td>60cm @ 85</td>
</tr>
<tr>
<td>F Roll out deck</td>
<td>1 Meter</td>
<td>1.5 Meter</td>
<td>2 Meter</td>
</tr>
<tr>
<td>D Drop in Area</td>
<td>Flat to 2 Meters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O Outside fence from</td>
<td>0.5 Meter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banners</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F Banner fence from wall</td>
<td>1.5 – 2 Meters</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 2.

Using halfpipe fields recommended by International Ski Federation (FIS), we will build mathematical models and verify these models with competition statistics. Also, we will analyze the influence of key parameters of each field on the performance of snowboarders, especially the distance of "vertical air," so as to establish the best shape for the halfpipe.

### 2. Assumptions :

A. Factors not related to the shape of the field

1. The athlete's gender, technical abilities, length of training and other physical and psychological
factors are not considered;
2. The athlete performs at an expected level during the course (e.g. no failure caused by slipping).
3. The influence of the snowboarding equipments are not considered.
4. The surface of the halfpipe (including the quality of snow) enables normal performance of the athlete.

B. Assumptions for the model
1. The sectional plane of the course is a straight line combined with two quarter arcs of standard ellipse.
2. The motion of the athlete in the air is regarded as a particle motion in a plane, which means twisting and flipping are in the same plane and he/she returns to the wall tightly.
3. The direction of the athlete flying out or coming back corresponds to the tangent of the curve.
4. Coefficient of friction is a constant, set as 0.03
5. The direction of speed is the same with that of the skate knife’s.
6. Air-resistance is negligible here.

3. Variable definition

Mass of snowboarder: $m$
Frictional work: $W_f$
Motion route: $S$
Positive pressure: $N$
Radius of curvature: $R = \frac{1}{k}$
Velocity: $v$
Width of halfpipe: $W$
Depth of halfpipe: $H$
In halfpipe velocity: $v_A$
Out halfpipe velocity: $v_D$
Altitude difference $\Delta h = \Delta h_{AB} + \Delta h_{BC} + \Delta h_{CD}$
Vertical air: $h$
4. Analysis of the Problem

During a snowboarding competition, one performance is made up of several motions. Each motion is followed by landing on the flat from the last motion and includes: curve-down after landing, flat-ski on the bottom, curve-up on the transition, fly, the motion in the air and landing. Because of the similarities of each motion, our model treats one single motion as the basis of the following analyses.

One motion can be separated to two parts: in the U-ramp and in the air.

The part in the U-ramp, as in the figure, is from A to D. This part can be divided into three small processes, curve-down (A to B), flat-ski (B to C), and curve-up (C to D). In all of these processes, only gravity and friction do work. From the definition formula of friction, we know that in the flat-ski process the athlete will move in a straight line with constant acceleration, in that the net force vertical to the flat is constant and so friction is constant. However, during the processes of curve-up and curve-down, the normal pressure is a function of the radius of curvature, which is not an invariant. So the friction varies with time, which makes the motion too complicated to be calculated.

In order to simplify the calculation by friction during the curve-up and curve-down processes, the following is assumed: the friction is the average value of the friction at the beginning and ending of the curve, and the displacement of the motion is the length of the curve. Thus, the work is the product. Because of the conservation of energy, we get the following equations:

**Flat-ski (B to C):** Accurate
\[
\left\{ \begin{array}{l}
\frac{1}{2} m v_C^2 - \frac{1}{2} m v_B^2 = mg \frac{B}{W} \Delta h_{AB} - w_{f2} \\
K mg \cos \theta \frac{B}{W} = w_{f2}
\end{array} \right. \quad (1)
\]

**Curve-down (A to B):** Approximation
\[
\left\{ \begin{array}{l}
\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = mg \left( H + \frac{W-B}{2W} \Delta h_{h} \right) - w_{f1} \\
K \left( m \frac{v_A^2}{R_A} + m \frac{v_B^2}{R_B} + mg \cos \theta \right) \frac{W-B}{2W} = w_{f1}
\end{array} \right. \quad (2)
\]

**Curve-up (C to D):** Approximation
\[
\left\{ \begin{array}{l}
\frac{1}{2} m v_D^2 - \frac{1}{2} m v_C^2 = mg \left( \frac{W-B}{2W} \Delta h_{BC} - H \right) - w_{f3} \\
K \left( m \frac{v_C^2}{R_C} + m \frac{v_D^2}{R_D} + mg \cos \theta \right) \frac{W-B}{2W} = w_{f3}
\end{array} \right. \quad (3)
\]

So if the initial speed is available, the speed from anywhere and the approximate work by friction can be derived.
The following kinetic data are from an analysis of movements of some Chinese snowboarders during the half-pipe snowboard Championship held at Harbin Institute of Physical Education on January 6 and 7 by using the video diagnosis technology (APAS). The initial velocity, which is the velocity when the snowboard enters the halfpipe is plugged into our model. The results are as follow:

Parameters of Harbin Mao ershan halfpipe course:

Inclination (°) 17.0
Length of course (m): 150.0
Width of halfpipe (m): 18.0
Depth of halfpipe (m): 5.0

![Diagram of Harbin Mao ershan halfpipe course](image)

**Figure 3. Cross section of the course**

In the cross section, line segment AB an CD is an ellipse of $a = 6.5m$, $b = 5m$, line segment BC is a 5m long straight-line segment, according to the equation of circumference of ellipse, the arc length of ABCD is $5\pi + 8m$.

$$\Delta h = l \times \tan 30^\circ \times \sin 17^\circ = 4m$$

In order to get a faster out halfpipe velocity, Snowboarders must take a route that has an angle with the cross section. According to analysis and research, the angle is usually between $20^\circ$ and $40^\circ$. In our model, we take it as $30^\circ$ to research the shape of halfpipe.

**Chart 2. Parameters of some good snowboarders[2]**

<table>
<thead>
<tr>
<th>player</th>
<th>weight</th>
<th>Velocity of gravity center when snowboard enters into halfpipe</th>
<th>Velocity of gravity center when snowboard gets out of halfpipe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54.0</td>
<td>10.50</td>
<td>11.65</td>
</tr>
<tr>
<td>2</td>
<td>71.2</td>
<td>9.04</td>
<td>11.39</td>
</tr>
<tr>
<td>3</td>
<td>58.4</td>
<td>12.51</td>
<td>13.73</td>
</tr>
<tr>
<td>4</td>
<td>58.9</td>
<td>10.62</td>
<td>11.20</td>
</tr>
<tr>
<td>5</td>
<td>58.9</td>
<td>10.82</td>
<td>12.00</td>
</tr>
</tbody>
</table>
Taking the parameters of these good snowboarders into our model, and using equation ①②③, we get the out halfpipe velocity and corresponding frictional work $W_f$ consumed in the process.

**Chart 3.**

<table>
<thead>
<tr>
<th>player</th>
<th>Velocity of gravity center when snowboard gets out of halfpipe</th>
<th>Curve-down (A to B) Frictional work $W_{f1}$</th>
<th>Flat-ski (B to C) Frictional work $W_{f2}$</th>
<th>Curve-up (C to D) Frictional work $W_{f3}$</th>
<th>total frictional work $W_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.97</td>
<td>510.82</td>
<td>115.46</td>
<td>596.4</td>
<td>1222.68</td>
</tr>
<tr>
<td>2</td>
<td>10.98</td>
<td>563.43</td>
<td>152.22</td>
<td>692</td>
<td>1407.65</td>
</tr>
<tr>
<td>3</td>
<td>13.43</td>
<td>698.8</td>
<td>124.86</td>
<td>770.4</td>
<td>1594.06</td>
</tr>
<tr>
<td>4</td>
<td>12.06</td>
<td>565.3</td>
<td>125.93</td>
<td>657.4</td>
<td>1348.63</td>
</tr>
<tr>
<td>5</td>
<td>12.2</td>
<td>578.9</td>
<td>125.93</td>
<td>669</td>
<td>1373.83</td>
</tr>
</tbody>
</table>

According to theorem of kinetic energy, using the data of the snowboarders in practice we can get the frictional work consumed in practice:

$$\frac{1}{2}m v_A^2 + mg \Delta h = \frac{1}{2} m v_D^2 + w_f$$

**Chart 4.**

<table>
<thead>
<tr>
<th>player</th>
<th>Out halfpipe velocity in theory</th>
<th>Out halfpipe velocity in practice</th>
<th>Frictional work in theory</th>
<th>Frictional work in practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.97</td>
<td>11.65</td>
<td>1222.68</td>
<td>1472.24</td>
</tr>
<tr>
<td>2</td>
<td>10.98</td>
<td>11.39</td>
<td>1407.65</td>
<td>1138.8</td>
</tr>
<tr>
<td>3</td>
<td>13.43</td>
<td>13.73</td>
<td>1594.06</td>
<td>1401.23</td>
</tr>
<tr>
<td>4</td>
<td>12.06</td>
<td>11.20</td>
<td>1348.63</td>
<td>1983</td>
</tr>
<tr>
<td>5</td>
<td>12.2</td>
<td>12.00</td>
<td>1373.83</td>
<td>1563</td>
</tr>
</tbody>
</table>

Comparing the calculated value of the model with the value in practice, it is obvious that they have some differences. The reason why they have these differences is that every snowboarder has their own features in the motion. So the values in theory and in practice basically follow the same regular pattern. In this aspect, we basically test and verify our model to be reasonable.

Model in the air:
In the process of motion in the air (from d to f), we ignore the air resistance, this process can be regarded as an ideal projectile motion. During the procedure, the only force is the gravity.

The motion can be decomposed into two components: one has an initial speed of $V_d \sin \alpha$ along Z, and has an acceleration of $g \cos \theta$ along Z; the other one has an initial speed of $V_d \cos \alpha$ along X, and has an acceleration of $g \sin \theta$ along X.

$$h_{\text{max}} = \frac{v_0^2 \sin^2 \alpha}{2g \cos \theta}$$

After the analysis of the model, we know that for a certain $\alpha$ and $\theta$, we may get the maximum out halfpipe velocity.

### 5. The Model Results

Based on the model we can maximize the vertical air by changing the shape of the course. As many parameters of the course will influence the vertical air, we apply control variable method to study this problem. We change one parameter of the course meanwhile keep others constant to get a higher out halfpipe velocity.
Figure 5. Inclination: 10°, 15°, 17°, 20°, 25°

Figure 6. Bottom flat: 3 m, 4 m, 5 m, 6 m, 7 m
Figure 7. Width 16, 17, 18, 19, 20

Figure 8. Height 4, 4.5, 5, 5.5, 6
We can conclude from above:

As the inclination and height of the course increase, the “vertical air” of snowboarders increases remarkably.

As the bottom flat and width of the course increase, the “vertical air” of snowboarders increases slowly.

Besides the requirement of “vertical air”, there are other requirements influencing on the performances of the athletes in the snowboarding competition, such as the execution, the degree of difficulty and the watchability of the tricks and the smooth connection between the actions. A great fraction of the tricks, which contain inverting, rotation, gliding, grabbing the snowboard and so on, are composed of the twist of body in the air. As a result, optimizing the requirement of obtaining the maximum twist in the air is also very important to improve the performance.

The twist in the air is composed of two parts.

One part is produced by the action of the athletes in the air, such as the rotation of the upper part of the body leading to the rotation of the whole body, the changement of the center of mass, and the adjustment of the moment of inertia. This part is not analyzed in this article, because it is almost determined by the technical skills of the athletes.

The other part is produced by the interaction between the athletes and the wall just before taking off the halfpipe, while the interaction is the torque caused by the friction in essence.

Taking the practical out halfpipe velocity into consideration, the interaction time of friction, by which the torque is provided, is very short. So we assume that the interaction time is tiny and constant.

\[ J \omega = ft \]

Obviously, the greater the friction is, the larger torque it can provide and the larger angular velocity the athlete gains.

As a common sense of snowboarding, the angle of the edge of a snowboard is the main factor of controlling the velocity and changing its direction. It proves that the angle of the edge determines the coefficient of friction: the angle is positive related to the coefficient of friction. To get larger stress, the athlete needs to increase the angle just before taking off. However, the more slanted the edge of the ramp performs to be, the more easily the athlete increases the angle of the edge. Therefore, in the point of the view of optimizing the shape of the snowboard course to obtain the maximum twist in the air, the edge of the ramp should be changed into a short tangent of the ellipse arc.

The stress has a great influence on the friction, and a great fraction of the stress is provided by the centripetal force. But the more slanted the tangent is, the bigger the radius of curvature is and
the smaller the stress is.

In conclusion, the angle of the tangent should be suitable to satisfy the two requirements. Analysis as follow:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \begin{cases} x = a \cos \theta \\ y = b \sin \theta \end{cases}
\]

One point lies in the ellipse, its tangent line is
\[
\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1
\]

\[
|\tan \beta| = \left| \frac{x_0 b^2}{y_0 a^2} \right| = \left| \frac{b}{a \tan \theta} \right|
\]

Radius of curvature
\[
R = \left( \frac{a^2 \sin^2 \alpha + b^2 \cos^2 \alpha}{ab} \right)^{\frac{3}{2}}
\]

So
\[
R = \left( \frac{1 + \tan^2 \beta}{b^2 + a^2 \tan^2 \beta} \right)^{\frac{3}{2}} a^2 b^2
\]

**Chart 5.**

<table>
<thead>
<tr>
<th>R/b</th>
<th>a=1.2b</th>
<th>a=2b</th>
<th>a=3b</th>
</tr>
</thead>
<tbody>
<tr>
<td>β =90</td>
<td>0.8333</td>
<td>0.5000</td>
<td>0.3333</td>
</tr>
<tr>
<td>85</td>
<td>0.8362</td>
<td>0.5043</td>
<td>0.3367</td>
</tr>
<tr>
<td>84</td>
<td>0.8375</td>
<td>0.5062</td>
<td>0.3382</td>
</tr>
<tr>
<td>83</td>
<td>0.8390</td>
<td>0.5085</td>
<td>0.3400</td>
</tr>
<tr>
<td>82</td>
<td>0.8408</td>
<td>0.5111</td>
<td>0.3421</td>
</tr>
<tr>
<td>81</td>
<td>0.8428</td>
<td>0.5141</td>
<td>0.3445</td>
</tr>
<tr>
<td>80</td>
<td>0.8450</td>
<td>0.5157</td>
<td>0.3472</td>
</tr>
</tbody>
</table>
Obviously, in a certain ellipse, the radius of curvature increases as $\beta$ decreases, and the increment degree of the radius of curvature with the decreasing angle is larger as the ratio of the major axis to the minor axis increases. Referring to the standard, the maximum ratio of the major axis to the minor axis is close to 3. It is valid that the angle is specified to be 85 degree, because the difference is controlled to the extent of 1%.

To keep the situation that the stress is provided by the centripetal force, the center of the mass should stay on the ellipse arc, not on the tangent. The minimum length of the snowboard $l$ is 129cm according to the references, and assume $\alpha = 60$ as before, then

$$\frac{l}{2} \sin \alpha = 55\text{cm}$$

### 6. Improve the Model

What tradeoffs may be required to develop a “practical” course?

1. Safety

   If the velocity out halfpipe is too big, the force of impact when landing will be a great shock to the athlete. Generally speaking, human beings can bear an impact force of 3–4 times of their weights. Now we calculate:

**Landing on a sloping surface [3]**

We consider a snowboarder dropping off a ledge onto a slope that is inclined at an angle $\theta$ to the horizontal as shown in figure 3. The snowboarder does not come to rest after landing on the slope because the component of the velocity parallel to the slope is not reduced to zero. Only the component of the velocity perpendicular to the slope is reduced to zero on landing. Since this is a two-dimensional problem we distinguish between the $x$ and $y$ components of vectors with a subscript. Once more the speed when the snowboarder first touches the ground is $v_y = -\sqrt{2gh}$. We now switch to the rotated $x'-y'$ coordinate system shown in figure 3 for convenience. The component of the velocity normal to the slope is along $y'$ and is given by

$$v'_y = v_y \cos \theta = -\sqrt{2gh \cos \theta}.$$  

We assume once more that during the landing the snowboarder’s center of mass drops a vertical distance $b$ as the snowboarder’s knees bend and that the final velocity $v'_{y'}$ along $y'$ is zero. We also assume the snow is hard packed and smooth so that there is no significant frictional force.

The acceleration $a'$ along $y'$ is
\[
\dot{a} = \frac{v_{f1}^2 - v_{y}^2}{2(-b) \cos \theta} = \frac{v_{y}^2}{2b \cos \theta}
\]

Application of Newton’s second law along \( y \) gives \( F_N - mg \cos \theta = ma \), which yields

\[
F_N = m(a' + g \cos \theta) = m\left(\frac{v_{y}^2}{2b \cos \theta} + g \cos \theta\right)
\]

Substituting for \( v' \) gives

\[
F_N = mg(1 + \frac{h}{b}) \cos \theta
\]

for the value of the normal force on landing.

Figure 10.

For human beings, we can only bear an impact force of 5–6 times of our weights. So, using

\[
F_N = 6mg
\]

Taking \( b \) is approximately 0.5 m.

For \( \theta \), normally it can be \( 60^\circ \), as it is shown as follow:
Using equation (1), we obtain:
\[ h = 5.5\text{m} \]

That means the vertical component of the velocity out half-pipe \( v_v \) is:
\[ v_v = \sqrt{2gh} = 10.49 \text{m/s} \]

As it is shown in the image below, we can take \( \alpha = 45^\circ \) generally.

### 7. Conclusions

Analyzing all the parameters of the halfpipe prototype to value their influences on the “vertical air” based on the model, we obtained, without considering other requirements, that “vertical air” increases as the inclination and depth of the halfpipe increase and the width and bottom plat have little influence on the “vertical air”. In order to satisfy the requirement of providing the maximum twist for the athletes, we optimized the halfpipe prototype and found that the edge of the wall performs to be a short beeline strengthening the interaction between the athletes and the wall to increase the initial angular velocity. As a result, the parameters of the beeline can be calculated and are close to the standard of FIS. Finally, considering an economic perspective and whether the athletes’ body can withstand impact when they rushing into the halfpipe and landing safely, we analyzed it needs to pursue maximum “vertical air” while taking consideration of speed limit. Therefore, we evaluated that the maximum “vertical air” is 6m in practical motion, when the inclination reaches 19°.
8. Strengths and weaknesses

Weaknesses
(1) We assume that the direction of velocity is the same with that of the skate knife’s. In fact, there often exist angles between them. The angle causes the loss the velocity.
(2) We assume that the motion of the athlete in the air is regarded as a particle motion in a plane. However, the edge of the ramp is not vertical. In fact, the angle is 85°; so the practical motion is dimensional but not in a plane. That causes certain errors.
(3) In the model, we ignore the air resistance, which will have some influence to the motion. So the calculated “vertical air” is higher than that in theory.

Strengths
(1) our model is precise. The calculating results correspond to the fact well.

References