Should Space Shuttles Simply Stop Serving?

2011HiMCM
Team #2973
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1. Introduction

The last flight mission of space shuttle marked the end of the score long space shuttle program. From 1981 to 2011, NASA’s six space shuttles have carried out 135 missions all together. With a glorious tradition, the end is not a farewell but a restart ushering in a new era. Yet, during this in-between time of the old and new programs, NASA has to rely on alternative ways to send its astronauts to ISS (International Space Station). This means a whole different budget plan and scenario for NASA.

Currently, there are several foreseeable plans. NASA can expedite its own plan for a new generation of space shuttle; or it may choose to depend on other nations and institutions for future transportations of astronauts and cargos. And other plans are also feasible. So it is our mission to help NASA pick out the best plan in order to maintain ISS and continue space probing programs in the next decade.

In our paper, we wish to develop a feasible flight plan with the largest profits. A cost and benefit analysis will be conducted to achieve our goal. We keep the number of astronauts and the amount of cargos sent as our decision variables in our model, and both the cost and the return function should be expressed based on these two variables.

To determine the relationship between annual return and the two variables, we make use of the past data of NASA. Firstly, we need to find out the annual income of the past few years. This part of the model takes on an economic approach, using the economic conception of investment return rate to evaluate a plan’s outcome. Two sub models are developed to figure out both the total income over years and annual income of a single project. Though the economic return is not a space program’s direct feedback, it is nevertheless an overall effect that a space program exerts on the society as a whole. We believe the social benefit of a space program is the key determinant, because the prime purpose of any scientific research projects is to serve the human race as a whole. This idea is also supported by NASA’s own
mission statement. Finally, regression is done to help us figure out the relationship with astronauts and cargos sent.

Besides the annual return model, we also set up a model for evaluating the annual cost of a candidate plan. We define the cost of a possible space program as composed by three separate parts: the cost of transporting necessary provisions and the cost of sending astronauts, and the maintenance cost of ISS. In order to take various circumstances into consideration, we basically divide the later model into separate discussions based on 2 time periods, as number of supplier changes as time goes by. The final comprehensive cost function is an integration of the above mentioned three parts and two time periods. Based on this comprehensive function, we make further revisions. There will be restrictive conditions regarding range of astronauts, cargos, flight frequencies, etc. Taken all these conditions into consideration and making necessary revisions, the cost model is completed.

Based on these two models, we are able to determine the how many people and cargo to send each year through a process of profit maximization. Hence, a completed plan for the next decade is derived by using these two models.

Though by no means a perfect schedule, this ten-year plan is both comprehensive and practicable. We believe it will be easy for NASA and other administrative organizations to adopt this plan as guidance. We also believe that, under such an all-rounded modeling and evaluation, the outcome should be a reasonably desirable answer for the question we have put forward at the beginning. However, the plan is also not without its problems: for example, the plan’s complexity increases its vulnerability and sensitivity for even a minor digression.
2. Assumptions

1. Our globe is not going to come to an end by the year 2012.

2. Russia is now willing to send astronauts and cargos for NASA at a price of $60 million per person and $7500 per pound.

3. Russia will be the only supplier until the year 2015. By then there will be other countries or individuals who can as well master such technology.

4. NASA will choose its suppliers only based on prices, excluding political factors or international relationships.

5. All spacecrafts provided by countries or institutions are considered the same machines only different in prices.
3. Model 1 – Annual Return Model

3.1 Introduction

In this model, we aim to evaluate that how much NASA, through investing huge amount of money on ISS since its emergence in 2000, can profit annually from the economic returns generated from the various projects implemented on ISS, and what’s its relation with space flights. To achieve such goal, we divided the problem into three sub models, each of which deal with a different problem regarding the questions we mentioned above.

Sub model 1 is focused on how long does it take for a project to gain its total profits, and how are the profits distributed according to time. Thus, we will be able to know how much we can get from a single project in a particular year. Sub model 2 discusses the ratio between cost and returns over a period of time. This model tends to solve the problem of how much is research and development weighted in a couple of elements that are related to outputs. By combining data of three sample cases, we can approximate the return ratio of the NASA research projects. Finally, sub model 3 will integrate the outcomes of the previous two, as the current year income can also be contributed by the researches done in the previous years. The former results can indicate that how much a typical research project can yield and in which proportion can be returned to a particular year. Hence, as long as we know how much projects or how many inputs will be devoted to International Space Station, we can easily tell the annual income of NASA dedicated by ISS, as well as its relation to people and goods sent.

Now let’s get started from the first sub model.
3.2 Sub model 1

This part establishes the return rate function of a single project. The return rate function is defined as a function which estimates the return rate on an annual basis. Based on past experiences and examples, it is assumed that the return rate value follows mountain-like distribution pattern during the span of a project. Such pattern, if true, reveals that the return rate of a project experiences a steady increase before enjoying its peak around the middle section, and later on sees a steady decrease before ends for good. Generally speaking, past research projects follow similar patterns with slight disparities, so it is proper to assume this kind of pattern appropriate for the space program.

To see if our assumption is true, we collected sample data from quarter sales of iPhone. The data we use range from 2007 to 2010, and focus on only one project. The influence of the sample project is similar to that of a space program, and the cost and economic return of the sample both share great resemblance. Thus, the sample should be a perfect match for a space program in terms of economic scale.

The sample data is a collection of quarterly sales revenue. Sales revenue is an indicator of how the economic return of a research program will behave, which fits into the purpose of this model.

All data collected is showed in the following chart and Graph 3.21.

<table>
<thead>
<tr>
<th>Time</th>
<th>Q3 07</th>
<th>Q4 07</th>
<th>Q1 08</th>
<th>Q2 08</th>
<th>Q3 08</th>
<th>Q4 08</th>
<th>Q1 09</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units Sold</td>
<td>270</td>
<td>119</td>
<td>2315</td>
<td>1703</td>
<td>717</td>
<td>6892</td>
<td>4363</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Q2 09</th>
<th>Q3 09</th>
<th>Q4 09</th>
<th>Q1 10</th>
<th>Q2 10</th>
<th>Q3 10</th>
<th>Q4 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units Sold</td>
<td>3793</td>
<td>5208</td>
<td>7367</td>
<td>8700</td>
<td>8750</td>
<td>8400</td>
<td>14100</td>
</tr>
</tbody>
</table>

Chart 3.21
Notice that a typical fiscal year of Apple starts at September each year, indicating the commencement for quarter 1.

From Graph 3.21, which we derived from the quarter sales of iPhones from Apple, it is clear that three separate distribution patterns exist: the first ranges from the fourth quarter of 2007 to the third quarter of 2008, the second ranges from the third quarter of 2008 to the second quarter of 2009, and the third ranges from the second quarter of 2009 to the third quarter of 2010. The three separate normal distribution patterns do not show mutual interferences, which indicates that the three patterns are caused by different incentives. This is supported by the fact that every in-between period of two consecutive normal distributions witnesses the born of a new product (iPhone 3G on quarter 3, 2008; iPhone 3Gs on quarter 2, 2009; iPhone 4 on quarter 3, 2010). As a result, we would like to discuss models based on three separate periods.

The fact also proves that, for one single research, its economic return strictly
follows certain distribution, where a program yields its greatest economic return during the middle period while the beginning and the ending sections are relatively weaker in terms of economic return. Such conclusion leads us to three assumed functions below that might match the graph we attained from reality.

**Assumed function No.1 - Normal Distribution Pattern**

In a normal distribution case, the function and graph go as followed:

\[
f(x) = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} + c
\]

![Graph 3.22](image)

This is an example of a typical normal distribution curve. \( \mu \) refers to the x-value of the curve’s symmetry axis.

By the definition of normal distribution, the area below curve adds to 1, and the width and height of the mountain-like curve is defined by the two parameters, \( \mu \) & \( \sigma \). To match the curve Graph 3.22, we would like to add a coefficient to the standard function above, and work out \( \mu \) & \( \sigma \) for the separate period through regression based on the data we got.

Here are the results:
Regressed Results for Normal Distribution Pattern

<table>
<thead>
<tr>
<th>Time period</th>
<th>$\sigma$</th>
<th>$\mu$</th>
<th>$C$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period 1</td>
<td>0.44858</td>
<td>2.39000</td>
<td>0.28000</td>
<td>0.14076</td>
</tr>
<tr>
<td>Time period 2</td>
<td>0.39179</td>
<td>2.37000</td>
<td>0.34000</td>
<td>0.15904</td>
</tr>
<tr>
<td>Time period 3</td>
<td>1.12067</td>
<td>4.79000</td>
<td>0.65000</td>
<td>0.10802</td>
</tr>
</tbody>
</table>

Chart 3.22

Time period 1 refers to quarter 3 of 2007 to quarter 3 of 2008. Time period 2 lasts from then on until quarter 2 of 2009. And Time period 3 ends in quarter 3 of 2010. MSE here in the abbreviation of mean squared error which will be explained later.

To give a more direct view of how much they are alike, let’s overlap the graph we derived from the normal distribution pattern onto the spots of the original one.
The curves with different colors here are derived from our results of regression in three time periods. Meanwhile, the green spots indicate the original data.

**Assumed function No.2- Polynomial pattern**

In this case, we will fit our inverted-U curve into a concave down parabola.

\[ f(x) = ax^2 + bx + c \]

And determine the coefficients of variables through regression, which leads to the outputs as shown below:
Regressed Results for Polynomial Pattern

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time period 1</strong></td>
<td>-0.34363</td>
<td>1.76920</td>
<td>-1.32160</td>
<td>0.05527</td>
</tr>
<tr>
<td><strong>Time period 2</strong></td>
<td>-0.24467</td>
<td>1.32054</td>
<td>-0.89448</td>
<td>0.11968</td>
</tr>
<tr>
<td><strong>Time period 3</strong></td>
<td>-0.03523</td>
<td>0.03609</td>
<td>0.07544</td>
<td>0.00215</td>
</tr>
</tbody>
</table>

Chart 3.23

Again, let’s see how closely it is related to our real graph,

Graph 3.24

The blue curve is the derived result of the polynomial pattern, while the red dots stand for the original data.

**Assumed function No.3- sinusoidal pattern**

Similar to the two cases above, function for this pattern commences with
\[ f(x) = a \cdot \sin(bx+c) + d \]

And finally lead to

| Regressed Results for Sinusoidal Pattern |
|----------------|---------|---------|---------|---------|
|               | a       | b       | c       | d       | MSE     |
| Time period 1 | 0.48000 | 2.09381 | 2.90000 | 0.64000 | 0.09723 |
| Time period 2 | 0.39000 | 2.09381 | 3.20000 | 0.65000 | 0.16651 |
| Time period 3 | 0.35445 | 0.60549 | -1.37493| 0.67395 | 0.00045 |

Chart 3.24

Graph 3.25

Notice that in all three functions above, we used programming to work out the regression, in which we first give out a range for each variable like a, b, c, etc. The
next step we take is to search all the combination of the values of the variables to find out which combination lead to the least MSE. Thus we can get the corresponding regressed function.

To figure out which of the three can be the fittest one of all, we would like to find out a method of comparison. While simple visual inspections on graphs can be way too inaccurate, we calculated the mean squared error (MSE) between each of the curve with the original one, which is to compute and average the square of the differences between worked-out y value of each point (indicating units sold) and the real one, as demonstrated by the formula in which $f(i)$ represents the regressed function and $y_i$ represents the original data.

$$MSe = \sqrt{\frac{\sum_{i=1}^{n} [f(i) - y_i]^2}{n}}$$

Integration of all the three graphs lead to the result indicated in Chart 3.25.

<table>
<thead>
<tr>
<th>Mean Squared Error for Three Assumed Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MSE</strong></td>
</tr>
<tr>
<td>Time period 1</td>
</tr>
<tr>
<td>Time period 2</td>
</tr>
<tr>
<td>Time period 3</td>
</tr>
</tbody>
</table>

Chart 3.25

After all, we can see that assumed function No.1 or the normal distribution pattern has a larger standard deviation in all three periods, indicating that it is not a
good-enough model to take. On the other hand, while the parabola works better for period 1 and 2, the sinusoidal one has an edge on the last period from quarter 2, 2009 to quarter 3, 2010. Since the selling of iPhone4 is more typical, the sinusoidal model seems more reasonable.

To avoid such inconsistency, we decide to employ the third model, the sinusoidal one due to two reasons. Firstly, if we take an overall look on the whole curve of a parabola, though it might fits more to the points we got from data, the derivative of it equals 2ax+b. With a as a negative number, the derivative decreases as the value of x increases. The function will inevitably take a deeper slope as the y value is smaller and smaller, and eventually reach a negative value of y, which is surely improper for our model. While for the sin curve, it tends to become flatter and smoother as it is reaching the minimum point. What’s more, as we have mentioned before, the third model is more suitable for the most recent period, which tells that the trend of a return rate cycle might be changing, and the sin pattern can be the most appropriate one to use today.

So after all, here is the final return rate model for a project.

\[ f(x)=a \cdot \sin(bx+c)+d \]

### 3.3 Sub Model 2

This part of modeling takes on a different approach which estimates the gross economic return based on input. The ultimate function will enable us to estimate the economic revenue by using the initial input. This requires a careful analysis of total costs and total revenue and the relation between cost and revenue. The basic rationale is stated as: first to determine the components of total costs; second to
define total costs; third to define total revenue based on total costs, which results in a formula clearly showing the relation between cost and revenue.

The first part of problem has already been addressed in the introduction part. The total costs are divided into: the cost of sales, the cost of research and development, the cost of marketing, and general and administrative cost. These four components are represented by $C_1$, $C_2$, $C_3$, and $C_4$ respectively.

The second part of problem is also straightforward. The total cost is defined as:

$$TC = C_1 + C_2 + C_3 + C_4$$

The third part is the core component. In order to curtail the complexity of modeling, we first have to prove that the relation between cost and revenue can be shown as a linear function and that the multiplier is in fact the return rate.

Suppose the relation between total revenue and cost components is not linear. It is proper to assume that the nonlinear function can be substituted by a linear approximation by introducing Taylor polynomial. Thus any nonlinear components can be transformed into a linear expression, and the ultimate function can be expressed in a linear function.

Thus, total revenue can be expressed as:

$$TR = a_1C_1 + a_2C_2 + a_3C_3 + a_4C_4$$

In which:

- $a_i$ represents the multiplier for every $C_i$
- $C_i$ represents the components of total costs

According to the above stated formula, we can also prove that the multiplier is the return rate. Suppose that $C_i$ is increased by $t$. Then the total revenue is increased by $a_i.t$; when $t$ equals one unit, $TR$ is increased by $a_i$. This indicates that, for every one
unit input, the increase of TR is ai; since the return rate is defined as:

**Return rate = the increase of TR / the increase of input**

, then the return rate is expressed as ai.

Thus, the two proof are completed. Based on the conclusion, we are able to adopt regression to determine the value for every ai.

Since the investment on NASA, or the ISS program, can mainly result in industry innovation and technology advancement, to stimulate such return on an economic sense, we adopted 3 different companies to analyze including Google, Cisco, and Apple. As Google focuses on software and service, Apple lays emphasis on innovation and products, and Cisco is about technology, we believe an overall evaluation of these three cases can deduce a return rate similar to the one in reality since they cover most of the fields that ISS research results will impact.

Data from three different projects are deprived and the range of the database spans from 2002 to 2010. Linear regression is adopted, and the result is both shown by the formulas and Chart 3.31-3.33.
<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>232</td>
<td>894</td>
<td>2070</td>
<td>4032</td>
<td>7056</td>
</tr>
<tr>
<td>Revenue</td>
<td>440</td>
<td>1466</td>
<td>3189</td>
<td>6339</td>
<td>10605</td>
</tr>
<tr>
<td>2007</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
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<tr>
<td>2009</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Cost of Revenue      | 6649  | 8622  | 8844  | 10417 |
| Research & Development| 2120  | 2793  | 2843  | 3762  |
| Sales & Marketing    | 1461  | 1946  | 1984  | 2799  |
| General & Administrative | 1279  | 1803  | 1668  | 1962  |
| Total Cost           | 11509 | 15164 | 15339 | 18940 |
| Revenue              | 16594 | 21796 | 23651 | 29321 |

Chart 3.31

Cost and Revenue of Cisco from 2002 to 2010

( in million dollars)

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Revenue</td>
<td>3448</td>
<td>3135</td>
<td>3192</td>
<td>3322</td>
<td>4067</td>
</tr>
<tr>
<td>Research &amp; Development</td>
<td>4264</td>
<td>4116</td>
<td>4530</td>
<td>4721</td>
<td>6031</td>
</tr>
<tr>
<td>Selling, General &amp; Administrative</td>
<td>618</td>
<td>702</td>
<td>867</td>
<td>959</td>
<td>1169</td>
</tr>
<tr>
<td>Total Cost</td>
<td>8330</td>
<td>7953</td>
<td>8589</td>
<td>9002</td>
<td>11267</td>
</tr>
<tr>
<td>Revenue</td>
<td>18915</td>
<td>1466</td>
<td>3189</td>
<td>20853</td>
<td>23917</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>2008</td>
<td>2009</td>
<td>2010</td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>Cost of Revenue</td>
<td>4598</td>
<td>5325</td>
<td>5208</td>
<td>5273</td>
<td></td>
</tr>
<tr>
<td>Research &amp; Develop</td>
<td>7401</td>
<td>8690</td>
<td>8403</td>
<td>8716</td>
<td></td>
</tr>
<tr>
<td>Selling, General &amp; Administrative</td>
<td>1151</td>
<td>1387</td>
<td>1565</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td>Total Cost</td>
<td>13150</td>
<td>15402</td>
<td>15176</td>
<td>15988</td>
<td></td>
</tr>
<tr>
<td>Revenue</td>
<td>34922</td>
<td>39540</td>
<td>36117</td>
<td>40440</td>
<td></td>
</tr>
</tbody>
</table>

Chart 3.32

The two items, sales & marketing, general & administrative is combined into one item, selling, general & administrative here based on the way they are presented in the year reports.

---

Cost and Revenue of Apple from 2002 to 2010

( in million dollars)

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Revenue</td>
<td>4139</td>
<td>4499</td>
<td>6022</td>
<td>9889</td>
<td>13717</td>
</tr>
<tr>
<td>Research &amp; Development</td>
<td>446</td>
<td>471</td>
<td>491</td>
<td>535</td>
<td>712</td>
</tr>
<tr>
<td>Selling, General &amp; Administrative</td>
<td>1111</td>
<td>1212</td>
<td>1430</td>
<td>1864</td>
<td>2433</td>
</tr>
<tr>
<td>Total Cost</td>
<td>5696</td>
<td>6182</td>
<td>7943</td>
<td>12288</td>
<td>16862</td>
</tr>
<tr>
<td>Revenue</td>
<td>5742</td>
<td>6207</td>
<td>8279</td>
<td>13931</td>
<td>19315</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>2008</td>
<td>2009</td>
<td>2010</td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td><strong>Cost of Revenue</strong></td>
<td>15852</td>
<td>21334</td>
<td>23397</td>
<td>39541</td>
<td></td>
</tr>
<tr>
<td><strong>Research &amp; Development</strong></td>
<td>782</td>
<td>1109</td>
<td>1333</td>
<td>1782</td>
<td></td>
</tr>
<tr>
<td><strong>Selling, General &amp; Administrative</strong></td>
<td>2963</td>
<td>3761</td>
<td>4149</td>
<td>5517</td>
<td></td>
</tr>
<tr>
<td><strong>Total Cost</strong></td>
<td>19597</td>
<td>26204</td>
<td>28879</td>
<td>46840</td>
<td></td>
</tr>
<tr>
<td><strong>Revenue</strong></td>
<td>24006</td>
<td>32479</td>
<td>42905</td>
<td>65225</td>
<td></td>
</tr>
</tbody>
</table>

Chart 3.33

Above is the data we collected from the three companies’ annual report, indicating the variables including C1 (Cost of Revenue), C2 (Cost of research and development), C3 (Cost of Sales), and C4 (General and Administrative). Since C3 and C4 are not decisive elements in our model, we allow them to integrate into one variable C5, as written in the original report of Apple and Cisco.

Through regression, we got the formula of TR (total revenue) of each of the three companies as indicated below:

**Google:** \( TR = 2.081 \times C1 + 4.383 \times C2 + 0.53006 \times C3 - 5.2161 \times C4 - 45.581 \)
Graph 3.31

The larger red dots stand for the original, while the smaller blue one represents the regressed result. The graph indicates high similarity.

Cisco: \[ TR = -1.9936C_1 + 6.0572C_2 - 2.2005C_5 + 562.83 \]
Graph 3.32

Apple: $TR = 0.88966 \times C1 + 21.393 \times C2 - 0.16124 \times C5 - 7220.3$
Notice that, among the four factors, the most important and most relevant determinant is the cost of research and development. Because the other factors are either minor determinants (general and administrative costs) or irrelevant parameters as to a scientific program (cost of sales and cost of marketing), it is fair to rule out those determinants and to lay focus on only one factor.

Also notice that C2 represents the cost of R&D (research and development), and the average multiplier for C2 is 10.6. This indicates that, for every unit input of R&D, total revenue is increased by 10.6 units, which we obtain through averaging the three coefficients of R&D. The result coincides with several academic papers we have consulted, in which the ratio of input and return ranges from 4 dollars to 14 dollars for every additional dollar invested in the space research program.

In all, according to our modeling, each dollar spent on research and development, or
in other words, projects and experiments on ISS, can lead to a total profit of 10.4 dollars which can be returned to NASA during several years.

3.4 Sub model 3

Finally, we come to the last sub model which helps to integrate the results of both of the previous ones. From its predecessors, we have already known that the return model of a single project follows a sinusoidal pattern, the cycle of which is evolved from the most recent curve of sales of iPhones.

\[ f(x) = a \cdot \sin(bx+c) + d \]

This is a graph of a typical sinusoidal curve with \( \lambda \) as the amplitude and \( T \) as the cycle.

Thus, we can present the coefficients in terms of \( \lambda \).
\begin{align*}
a &= \frac{\lambda}{2} \\
b &= \frac{2\pi}{T} \\
c &= -\frac{\pi}{2} \\
d &= \frac{\lambda}{2}
\end{align*}

The function can be written in the form of

\[ f(x) = \frac{\lambda}{2} \sin\left(\frac{2\pi}{T} x - \frac{\pi}{2}\right) + \frac{\lambda}{2} \]

Thus the total area between the function and the x-axis represents the total revenue.

\begin{center}
\textbf{Graph 3.42}
\end{center}

They area painted grey equals the total revenue generated by one product in a complete cycle.

Suppose the total revenue as the unit value of 1. Then the following equation is set up.

\[
\int_{0}^{T} \left[ \frac{\lambda}{2} \sin\left(\frac{2\pi}{T} x - \frac{\pi}{2}\right) + \frac{\lambda}{2} \right] dx = 1
\]

which can be solved with the following steps,
\[- \frac{\lambda}{2} \frac{T}{2\pi} \cos\left(\frac{2\pi}{T} x - \frac{\pi}{2}\right) + \frac{\lambda}{2} x\right]_0^T = 1

\[- \frac{\lambda}{2} \frac{T}{2\pi} \cos\left(\frac{2\pi}{T} T - \frac{\pi}{2}\right) + \frac{\lambda}{2} T + \frac{\lambda}{2} \frac{T}{2\pi} \cos\left(\frac{2\pi}{T} \cdot 0 - \frac{\pi}{2}\right) + \frac{\lambda}{2} \cdot 0 = 1

\[- \frac{\lambda}{2} \frac{T}{2\pi} \cos\left(\frac{3\pi}{2}\right) + \frac{\lambda T}{2} + \frac{\lambda}{2} \frac{T}{2\pi} \cos\left(- \frac{\pi}{2}\right) = 1

\frac{\lambda T}{2} = 1

Thus,

\[\lambda T = 2\]

Then we substitute \( \lambda = \frac{2}{T} \) with \( T \), and get the final function as

\[f(x) = \frac{1}{T} \sin\left(\frac{2\pi}{T} x - \frac{\pi}{2}\right) + \frac{1}{T}\]

By collecting data, we get the average of \( T=2.584241136 \) yrs. Then for the total R&D expenditure of a year, its revenue is returned in three years, with

\[
\int_0^1 \left[ \frac{1}{T} \sin\left(\frac{2\pi}{T} x - \frac{\pi}{2}\right) + \frac{1}{T} \right] dx = 0.2805705194
\]
\[
\int_1^2 \left[ \frac{1}{T} \sin\left(\frac{2\pi}{T} x - \frac{\pi}{2}\right) + \frac{1}{T} \right] dx = 0.6481474458
\]
\[
\int_2^T \left[ \frac{1}{T} \sin\left(\frac{2\pi}{T} x - \frac{\pi}{2}\right) + \frac{1}{T} \right] dx = 0.0712820348
\]

which means 28.06% of its revenue is returned in the first year, 64.81% in the second year, and finally 7.13% in the third year.

What’s more, we have also known inputs in research programs will lead to 10.6
times of returns either in terms of service or products. For each project, that amount of money will be obtained through 2.584241136 yrs.

That is to say, for each individual year n, the return can be calculated through the formula below:

\[ R_n = 10.61106667 \times (0.2805705194 \times C_n + 0.6481474458 \times C_{n-1} + 0.0712820348 \times C_{n-2}) \]

\( C_i \): Cost of Year \( i \)  \quad \( R_i \): Revenue of Year \( i \)

The formula above indicates that the NASA’s profit of one year from ISS is determined by 7.13\% of the return caused by projects done 2 years ago, 64.81\% of the return caused by projects done year ago, and 28.06\% of the return caused by projects done the current year.

To make our model more practical, instead of figuring out the cost of each project, we use annual cost as a combination of cost of projects for our calculations. Based on the data we got from NASA’s website, money spent on ISS, which we consider as input in Research and Development from 2004 to 2010 is shown in the chart 3.41.

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of R&amp;D</td>
<td>6109</td>
<td>6674</td>
<td>6053</td>
<td>6108.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of R&amp;D</td>
<td>6791.1</td>
<td>5764.7</td>
<td>6180.6</td>
</tr>
</tbody>
</table>

Chart 3.41
The data of “Cost of R&D” in the chart is equal to NASA’s expenditure on ISS as we generally consider all projects and activities done on ISS R&D payments.

Employing the formula and Chart 3.41, we can get the outcome for annual income from 2006 to 2010 as shown in Chart 3.42

<table>
<thead>
<tr>
<th>Year</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>68542</td>
<td>64863</td>
<td>66807</td>
<td>68489</td>
<td>63184</td>
</tr>
</tbody>
</table>

Chart 3.42

After computing the revenue of NASA generated from ISS in the past 5 years, we are able study its relationship with the frequency and payloads of spaceflights. To facilitate our model, we assume that number of astronauts, amount of equipments and goods, and time as the only factors that will affect the research on ISS, which eventually lead to an impact on the annual income. But in accordance with our research, ISS has always been kept full-loaded with 6 persons ever since its establishment, which makes the factor, “time”, a constant one. Thus we would like to develop a function of Revenue involving the two factors, people (X) and cargos(Y), which goes as followed.

\[ R_i = (a_1 X_i + b_1)(a_2 Y_i + b_2) \]

\( R_i \) is the revenue earned in year I,

\( X_i, Y_i \) represents the number of people and cargos (in pounds) sent in year i
Using the data we generated from combining various flight records in a year, we come to the answer shown in Chart 3.43

<table>
<thead>
<tr>
<th>Flight and Payload Records of ISS from 2007 to 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="#" alt="Table" /></td>
</tr>
<tr>
<td><strong>X</strong> 2010 2009 2008 2007</td>
</tr>
<tr>
<td>19 34 28 21</td>
</tr>
<tr>
<td><strong>Y</strong> 61.425 110.51 93.416 69.01</td>
</tr>
</tbody>
</table>

Chart 3.43

The value of Y here has a unit of klbs

Finally, regressing the data above leads us to the final answer of the annual return model. The relationship between annual return and people and goods sent into space can be illustrated in the formula below.

$$ R_i = 132 * X_i + 26.35 * Y_i + 1.37 * X_i Y_i + 57145 $$

$R_i$ is the revenue earned in year $I$,

$X_i, Y_i$ represents the number of people and cargos (in pounds) sent in year $i$
4. Model 2 - Annual Cost Model

4.1 Introduction

After attaining the annual income generated by investment on ISS, in order to complete our Cost and Benefit Analysis, we now come to the annual cost spent on implementing projects on ISS.

Taking the various flight choices that might emerge several years later into consideration, we would like to divide the cost calculation into 2 time periods. The first one is from today to 2015. And the next is from 2015 to 2020, when NASA can choose to either use their own newly developed space shuttle, or foreign space ships provided not only Russia, but also other countries or individuals that can become suppliers by then. Through this way, we divide the annual total cost model into 2 sub ones.

Sub model 1

In period one, from now to 2015, the model is relatively simple. The variables here include the number of astronauts, or X, and weight of equipments and necessities, or Y. Since NASA has the only choice of sending people with the help of the Russians, the cost per person is fixed at $60 million. Based on the settled cost of person and goods, the annual total cost can be easily shown in with a function regarding number of astronauts and goods sent each year.

Sub model 2

The function of period two, which is more complicated, considers more facts. After the year 2015, there might be more countries other than Russia that can be able to provide space ships for NASA fights to ISS, comprising India, China, or the
European countries. In such cases, the prices charge for every astronaut will very likely be lowered, as it is no longer a case of monopoly, but that of an oligopoly. Here we will apply the economic knowledge of game theory to indicate such change.

Notice that another factor, the maintenance cost of ISS is taken into consideration in both of the cases. The cost of maintenance, as we assumed, should be related to both the length of service of ISS, and how frequently do the flights take place. On account of our common senses, it is reasonable to presume that the longer it works, the more frequently flights are sent, the higher the maintenance cost will be. Then, it is our job to find out the relationship between these three factors.

Now let’s get started from the simpler Sub model 1.

### 4.2 Sub Model 1

To simplify our formula, we first number the year 2011 to 2020 as year No.1 to Year No.10. According to our assumption stated before in the introduction, the annual total cost is determined by such factors containing number of people, and weight of goods sent, together with the maintenance cost of ISS. In terms of formula, it can be illustrated as below.

\[ TC = C_a + C_c + C_m \]

- \(C_a\): Cost of Astronauts
- \(C_c\): Cost of Cargo
- \(C_m\): Cost of Maintenance

1. \(C_a\)

   Assuming that the cost of sending an astronaut into the space will remain constant at 60 million dollars, we can easily deduct that \(C_a\) is a linear equation of \(X\), the number of astronauts that go into the space.
\[ C_a = k_1 \times X \]

\( k_1 \): Constant, 60 million dollars per astronaut

2. \( C_c \)

Bearing in mind that for every astronaut sent into space, a certain amount of daily necessities has to go up with them, we divide the cargo into two parts, cargos related to astronauts and others, shown in the formula as follows.

\[ C_c = k_2 \times (m_1 \times X + Y) \]

\( X \): Number of Astronauts \n\( Y \): Weight of other Cargo \n\( m_1 \): Constant, weight of cargo for one astronaut, calculated below \n\( k_2 \): Constant, cost of sending one pound of cargo into space, equals to 0.0075 million Dollars per pound

To calculate how much cargo is needed for one astronaut, we collect the data of 10 Space Shuttle Launches, reproduced below.

<table>
<thead>
<tr>
<th>Mission No.</th>
<th>STS 135</th>
<th>STS 134</th>
<th>STS 132</th>
<th>STS 131</th>
<th>STS 130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Astronauts</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Total Payload</td>
<td>28418</td>
<td>34760</td>
<td>26615</td>
<td>33800</td>
<td>39010</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mission No.</th>
<th>STS 128</th>
<th>STS 127</th>
<th>STS 126</th>
<th>STS 125</th>
<th>STS 123</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Astronauts</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Total Payload</td>
<td>37420</td>
<td>30080</td>
<td>38300</td>
<td>28890</td>
<td>37290</td>
</tr>
</tbody>
</table>

Chart 4.21
And the data thus leads to the following chart.

<table>
<thead>
<tr>
<th>No. of Astronauts</th>
<th>4</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Payload</td>
<td>28418</td>
<td>33461.7</td>
<td>34296.7</td>
</tr>
</tbody>
</table>

Chart 4.22

As mentioned before,

\[ TP = m_1 \times X + Y \]

TP: Total Payload

We use linear regression to get the value of \( m_1 \), which is equal to 1976.85753.

Here we finally take 2000 pound per person as the value of \( m_1 \).

3. \( C_m \)

By checking NASA’s budget for ISS from year 2011 to 2015, we can easily get the cost of maintenance for ISS as the following chart.

<table>
<thead>
<tr>
<th>Year</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISS Budget</td>
<td>2779.8</td>
<td>2983.6</td>
<td>3129.4</td>
<td>3221.9</td>
<td>3182.8</td>
</tr>
</tbody>
</table>

Chart 4.23

To sum up, the annual cost from year 2011 to 2015 can be expressed with the following formula:
4.3 Sub model 2

The main disparity between Sub model 2 and the former one is on the calculation of unit costs. In this case, we consider the situation between 2015 and 2020. By then, it is very likely that the US has already managed to develop its own spaceship. Meanwhile, there can also be other countries that can be qualified for sending American astronauts, which, as a matter of experience, will eventually lead to a descended price.

But to be more objective, we would like to illustrate such fact through an economic model. If we consider sub model 1 as an example of monopoly, where a specific person or enterprise is the only supplier of a particular commodity, sub model 2 satisfies the definition of an oligopoly, a market form in which a market or industry is dominated by a small number of sellers, or oligopolists. Also notice that due to very few competitors, each oligopolist is likely to be aware of the actions of the others. In our case, the market here refers to the opportunity of gaining profits from providing spaceships for NASA, while the oligopolists stand for countries with technology advancement in the near future, such as the European Union, East Asian countries, or maybe India some day.

Before our discussion of oligopoly, there is one circumstance that we want to exclude, the collusion case, where the countries we have mentioned before simply set agreements with each other (maybe secretly) and decide the price together. Such situation can probably happen in our real world, but to simplify our model, it is regarded as another example of monopoly equivalent to that of sub model 1.

Now let’s see what differences are between unit costs of sub model 1 and 2.
We’ll get started from case 1.

From 2011 to 2015, when Russia is the only supplier of NASA, the technology superiority makes the country a natural monopolist in the world market of spaceships. Similar to the monopolistic companies, it is reasonable to assume that Russia bears the characteristics of a price maker, the single seller, and most importantly, a profit maximizer. Since the ability of manufacturing spacecrafts of Russia today does not match the need for astronauts for 2 countries, we assume that the country will always be willing to produce as long as marginal cost is equal or less than marginal revenue, or in other words, unit price. Notice that the marginal cost is different concept from average cost. Marginal cost (MC) is defined as the money paid for every extra product manufactured. Usually, MC curve is a U-shaped one which is relatively stable or slightly downward sloping at the beginning, and goes on increasing soon after the quantity grows. This is basically because resources will be less efficiently utilized as the quantity increases. To avoid focusing too much on economic concepts, let us continue to see the model in Graph 4.31

Graph 4.31

Market with single seller

MR=Marginal Revenue;
Graph 2.1 illustrates a typical demand-supply relationship of a monopoly market. Due to its pricing power, the demand curve, which is equivalent to price, is settled above its costs. The model reached its equilibrium when MC=MR at point H, with an ATC of B, and a selling price of A, which is $60 millions. According to such model, Russia, in the first five years, is able to earn a profit equal to the area of rectangle ABFG through sending US astronauts.

But such benefits are not going to last forever. As time goes by, more countries are capable of providing spacecrafts for foreign countries, and all of them are willing to make profits out of such trades. Instead of a straight line of Demand, we will have something much different.

The market after 2015 can be best described as oligopolistic competition, since more commercial companies and countries may be able to provide space shuttles for the US astronauts. Traditionally, there are several few theories of describing such a market, including but not limited to Stackelberg Theory that deal with firms move sequentially, Cournot Theory that focuses on quantity competitions, and Bertrand Theory who discusses solely price decisions. Since we are now trying to figure out the impact of competition on prices, it’s appropriate for us make use of the Bertrand model. With few assumptions, we can apply the Bertrand model for oligopoly here. The assumptions involve:

1. NASA will trade with the one with the lowest price regardless of political factors,
2. Since countries and individuals today are fleetingly multiplying their capacity of manufacturing spaceships, we presume that they are able to support the total demand without capacity limitations,

3. Spacecrafts produced by all countries or individuals are the same.

In a Bertrand Model, the supplier with the lowest price win the whole market, which leads to fierce competition between oligopolists and drives the price down until to the marginal cost. Since the product, space shuttles is often produced at a small number, we can approximate marginal cost to the cost itself. Then it comes to the turn to calculate the future cost of sending astronauts and cargo into space.

Noticing the Moore’s Law for computer, which indicates the computers’ performance will double every 18 months, we find it possible that we can develop a similar law for space shuttles. We take the data of three space shuttle launches of China, Shenzhou 5th, Shenzhou 6th, and Shenzhou 7th.

<table>
<thead>
<tr>
<th>Cost per Astronaut of Shenzhou Spaceships (in billion Yuan)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Number of Astronauts</td>
</tr>
<tr>
<td>Year Launched</td>
</tr>
<tr>
<td>Cost of Launching</td>
</tr>
<tr>
<td>Cost per Astronaut</td>
</tr>
</tbody>
</table>

Chart 4.31

1 dollar approximately equals 6.4 Yuan.
By using these data, we finally get the result that for space shuttles, $p=5.6167$ yrs. This means that every 5.6167 years, the cost of launching space shuttles will become half the price as it used to be five years earlier. In another expression, every year, the cost of launching space shuttles will cut by 11.61%.

Applying to the 11.61% data to the year 2015 price, we can easily get the cost for the year 2016 to 2020 as indicated in the following Chart 4.32.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost per Astronaut</th>
<th>Cost per Pound of Cargo</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>60</td>
<td>0.0075</td>
</tr>
<tr>
<td>2016</td>
<td>53.034</td>
<td>0.00662925</td>
</tr>
<tr>
<td>2017</td>
<td>46.87675</td>
<td>0.005859594</td>
</tr>
<tr>
<td>2018</td>
<td>41.43436</td>
<td>0.005179295</td>
</tr>
<tr>
<td>2019</td>
<td>36.62383</td>
<td>0.004577979</td>
</tr>
<tr>
<td>2020</td>
<td>32.37181</td>
<td>0.004046476</td>
</tr>
</tbody>
</table>

Chart 4.32

For cost of maintenance of ISS, we use NASA’s budget for ISS maintenance from 2010 to 2015 to get the future cost of maintenance. The data is as follows. (Unit: million dollars)
### Maintenance Budget for ISS

(in million dollars)

<table>
<thead>
<tr>
<th>Year</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010(0)</td>
<td>2317</td>
</tr>
<tr>
<td>2011(1)</td>
<td>2779.8</td>
</tr>
<tr>
<td>2012(2)</td>
<td>2983.6</td>
</tr>
<tr>
<td>2013(3)</td>
<td>3129.4</td>
</tr>
<tr>
<td>2014(4)</td>
<td>3221.9</td>
</tr>
<tr>
<td>2015(5)</td>
<td>3182.8</td>
</tr>
</tbody>
</table>

Chart 4.33

The number beside the year represents its value in the functions later.

, which results in a concave-down curve as drawn in Graph 2.2.
Again, we use possible regression of sinusoidal function and polynomial function, we get two possible regressed functions reproduced below.

Sinusoidal:

\[ f(x) = 4466.46172 \sin(0.15707963x + 0.93463288) - 1244.625 \]

Polynomial:

\[ f(x) = -41.135714x^2 + 352.821428x + 2452.47714 \]

Noting that the MSE for the sinusoidal regression is 1466.63685 while the MSE for the polynomial one is only 169.770809, which is far smaller than that of the sinusoidal one, thus we choose the polynomial one as our regressed function.
In conclusion, the cost function for year 2016 to 2020 can be illustrated as followed in which n represents the year.

\[ TC = 0.8839^{n-5} \times [60 \times X + 0.0075 \times (2000 \times X + Y)] - 41.14n^2 + 352.82n + 2452.48 \]
5. Conclusion

After determining the value of return and cost, we can now complete our cost and benefit analysis by combining these two models to compute the profit (P).

\[ P_i = R_i - TC_i \]

\[ = (132 \times X_i + 26.35 \times Y_i + 1.37 \times X_i Y_i + 57145) - \]
\[ [60 \times X + 0.0075 \times (2000 \times X + Y) + C_m] (i = 1, 2, 3, 4, 5) \]

During the first five years. Or,

\[ = (132 \times X_i + 26.35 \times Y_i + 1.37 \times X_i Y_i + 57145) - \]
\[ \{0.8839^n \times [60 \times X + 0.0075 \times (2000 \times X + Y)] - 41.14n^2 + 352.82n + 2452.48 \} \]
\[ (i = 6, 7, 8, 9, 10) \]

during the later 5 years.

Using computer programming to achieve our profit maximization, we get the answer as shown in our final chart,

<table>
<thead>
<tr>
<th>Year</th>
<th>No. of Astronauts</th>
<th>Weight of Cargo (thousand lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>15</td>
<td>47</td>
</tr>
<tr>
<td>2012</td>
<td>15</td>
<td>47</td>
</tr>
<tr>
<td>2013</td>
<td>15</td>
<td>47</td>
</tr>
<tr>
<td>2014</td>
<td>15</td>
<td>47</td>
</tr>
<tr>
<td>2015</td>
<td>15</td>
<td>47</td>
</tr>
<tr>
<td>2016</td>
<td>16</td>
<td>54</td>
</tr>
<tr>
<td>2017</td>
<td>17</td>
<td>63</td>
</tr>
<tr>
<td>2018</td>
<td>19</td>
<td>75</td>
</tr>
<tr>
<td>2019</td>
<td>21</td>
<td>92</td>
</tr>
<tr>
<td>2020</td>
<td>25</td>
<td>112</td>
</tr>
</tbody>
</table>

Chart 5.1
This chart illustrates the FINAL answer of our paper!!! We believe through our modeling, it is both a feasible and profitable plan for NASA to use.
6. Strengths and Weaknesses

6.1 Strengths

A very obvious aspect of our paper is that it contains a lot of regression. By using various methods of regression, we are able to take numerous factors into consideration including the return rate, ISS maintenance cost, etc. Our model is an all-round model which includes a lot of factors.

By the mean time, our model is also easy to conduct. By giving out the final plan with each year’s number of astronauts and weight of cargo, we provide a most comprehensible and feasible plan for NASA. All it needs to do is to schedule these people and this amount of cargo into several launches.

Last but not least, our model has a very good compatibility. Since we use a lot of parameters during the computing process, one can easily change the value of one or more of the parameters to get the model adapted to the current situation. Thus our model can be used more efficiently.

6.2 Weaknesses

One possible improvement of our model might be specifying the future plan, determining on what day and where should NASA launch what number of astronauts and what amount of cargo. But due to the lack of specific information, we can only stop at what we have done, which includes the number of astronauts and weight of cargo each year.

Another possible improvement would be taking a more micro look at the model. Currently, our model is created in a macro way. It reflects the macro trend, but neglects those micro relationships between different factors. By taking a more micro look, we might make our model more specific and congruent to reality. However, it is again the lack of data and information that keeps us from doing so.
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3. www.wikipedia.com

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   http://en.wikipedia.org/wiki/Oligopoly
   http://en.wikipedia.org/wiki/Bertrand

4. www.apple.com (Year reports for Apple)

5. www.google.com (Year reports for Google)

6. www.cisco.com (Year reports for Cisco)

7. www.nasa.gov

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   China National Space Administration
8. Appendix

1. Regression of $R = a_1 C_1 + a_2 C_2 + a_3 C_3 + a_4 C_4$ (MATLAB Code)

Cisco:

$c1 = [3448 3135 3192 3322 4067 4598 5325 5208 5273]$;
$c2 = [4264 4116 4530 4721 6031 7401 8690 8403 8716]$;
$c3 = [618 702 867 959 1169 1151 1387 1565 1999]$;

$r = [18915 18878 18550 20853 23917 34922 39540 36117 40040]$;
format short g
$Y = r'$
$X11 = [ones(1,length(r));c1;c2;c3]'$
$B1 = regress(Y,X11)$

Apple:

$c1 = [4139 4499 6022 9889 13717 15852 21334 23397 39541]$;
$c2 = [446 471 491 535 712 782 1109 1333 1782]$;
$c3 = [1111 1212 1430 1864 2433 2963 3761 4149 5517]$;

$r = [5742 6207 8279 13931 19315 24006 32479 42905 65225]$;
format short g
$Y = r'$
$X11 = [ones(1,length(r));c1;c2;c3]'$
B1=regress(Y,X11)

Google:

c1=[132 626 1458 2577 4225 6649 8622 8844 10417];
c2=[32 91 226 600 1229 2120 2843 3762];
c3=[44 120 246 468 850 1461 1946 1984 2799];
c4=[24 57 140 387 752 1279 1803 1668 1962];
r=[440 1466 3189 6339 10605 16594 21796 23651 29321];

format short g

Y=r'

X11=[ones(1,length(r));c1;c2;c3;c4]'

B1=regress(Y,X11)

2. Regression of investment return function (C Code)

#include<stdio.h>

#include<math.h>

main()
{
    FILE *input, *output;
    float a[10],b[10],n,m,p,min,b2[10],sum,sa,sb,sc,q,sd;
    int c,i;
input=fopen("D:/input1.txt","r");

fscanf(input,"%i\n",&c);

for(i=1;i<=c;i++)
{
fscanf(input,"%f %f\n",&a[i],&b[i]);
}

fclose(input);

min=100000;

for(n=0;n<=1;n+=0.01)
{
    printf("%f\n",n*100);
    for(m=3.1415926/2;m<=3.1415926/3*2;m+=0.001)
    {
        for(q=-0.2;q<=1.2;q+=0.01)
        {
            for(p=0;p<=3.5;p+=0.1)
            {
                sum=0;
                for(i=1;i<=c;i++)
                {
                    b2[i]=n*sin(a[i]*m+p)+q;
                    sum+=(b2[i]-b[i])*(b2[i]-b[i]);
                }
                sum=sqrt(sum/(c));
                if (sum<=min)
                {
                    sa=n;
                    sb=m;
                    sc=p;
                    sd=q;
                }
            }
        }
    }
}
```c
min=sum;

}
}
}
}
output=fopen("D:/output1.txt","w");
fprintf(output,"%f %f %f %f %f",sa,sb,sc,sd,min);
fclose(output);
input=fopen("D:/input2.txt","r");
fscanf(input,"%i\n",&c);
for(i=1;i<=c;i++)
{
    fscanf(input,"%f %f\n",&a[i],&b[i]);
}
fclose(input);
min=100000;
for(n=0;n<=1;n+=0.01)
{
    printf("%f\n",n*100);
    for(m=3.1415926/2;m<=3.1415926/3*2;m+=0.001)
    {
        for(q=-0.2;q<=1.2;q+=0.01)
        {
            for(p=0;p<=3.5;p+=0.1)
            {
                sum=0;
                for(i=1;i<=c;i++)
```
{  b2[i]=n*sin(a[i]*m+p)+q;
    sum+=(b2[i]-b[i])*(b2[i]-b[i]);}
    sum=sqrt(sum/(c));
    if (sum<=min)
      {  sa=n;
          sb=m;
          sc=p;
          sd=q;
          min=sum;
      }
  }
}
}
}
}
output=fopen("D:/output2.txt","w");
fprintf(output,"%f %f %f %f %f",sa,sb,sc,sd,min);
fclose(output);input=fopen("D:/input3.txt","r");
fscanf(input,"%i\n",&c);
for(i=1;i<=c;i++)
  {fscanf(input,"%f %f\n",&a[i],&b[i]);
  }
fclose(input);
min=100000;

for(n=0;n<=1;n+=0.01)
{
    printf("%f\n",n*100);
}

for(m=3.1415926/3;m<=3.1415926;m+=0.001)
{
    for(q=-0.2;q<=1.2;q+=0.01)
    {
        for(p=0;p<=3.5;p+=0.1)
        {
            sum=0;
            for(i=1;i<=c;i++)
            {
                b2[i]=n*sin(a[i]*m+p)+q;

                sum+=(b2[i]-b[i])*(b2[i]-b[i]);
            }

            sum=sqrt(sum/(c));

            if (sum<=min)
            {
                sa=n;
                sb=m;
                sc=p;
                sd=q;
                min=sum;
            }
        }
    }
}

output=fopen("D:/output3.txt","w");

fprintf(output,"%f%f%f%f%f",sa,sb,sc,sd,min);

fclose(output);

3. Determining each years x,y value (C code)

#include<stdio.h>
#include<math.h>

main()
{
    long int x,y,xs,ys;
    long double r, max;
    max=0;
    for (x=1;x<=30;x++)
    {
        for(y=60000;y<=200000;y+=100)
        {
            r=0.001*(132*x+0.02635*y+0.00137*x*y+57145)-60*x-0.0075*(2000*x+y)-2.7798;
            if (r>=max)
            {
                max=r;
                xs=x;
                ys=y;
            }
        }
    }
}
printf("%ld %ld", xs, ys);
}