Our experience of everyday life tells us that, baking quantity of food is relevant to the shape of pan. So we build models aiming to find out this specific relationship.

We divide the pan in the oven into three parts: a Pan-&-Oven Heat Source, an Imagined-air media and an Above-air media. Each of these is also divided into small units. We quantize energy, and hypothesize it to be Energetic Particles which contain a fixed amount of energy only. The heat convection is out of consideration, which will simplify the study.

Model 1 takes only the influence of heat emission into consideration. Monte Carlo Method helps us to simulate the process of heat transfer, and thus gives us the number of particles an Imagined-air layer unit gets. In this section, we use DIALux to simulate the heat emission process. DIALux is a kind of software that can simulate the light in a room, and the light radiation has a lot similarity with heat radiation.

Model 2 analyzes s-conduction (a little different from heat conduction) only.

Model 3 is more comprehensive and more scientific since all the influencing factors we analyzed above are considered. A weight $\alpha$ is indicated to distinguish the importance of conduction and thermal radiation. We obtain a formula from the last model, so any shape of a pan can get its heat-distribution parameter by putting its roundness into it.

Model 3 generates a detailed description of heat distribution in a pan, and gets the relationship between the shape and the unevenness of heat distribution of the pan.

For the second question, since pans differ in shape and the ovens are mostly rectangular, the study only focuses on two most widely used sizes of ovens, and analyzes four kinds of fixed-area pans----an oblong, a quadrate, a round, and a specially-designed pan. Optimal laying out pattern, the analytical method used in model 3, and the formula we obtained above help us find out that, the pan we creatively designed is the best in most cases.

From our model, the effects of the four kinds of pans are generated. We design a new shape of pan named Ultimate Brownie Pan based on the results of the models, which performs better and can be more flexible.
Heat Distribution Models and the Ultimate Brownie Pan

Introduction

Facts and practice have all proved that, when baking food, round pan is the first choice (compared to other shapes). The most important reason why round pans are so widely used is that, heat can be distributed evenly over the entire outer edges and the products won’t be overcooked at the edges. But if we bake things in a pan which has a shape other than circle----a rectangular for example----heat is concentrated in the 4 corners and the food gets overcooked at the corners.

So, what’s the relationship between the shape of a pan and the distribution of heat of the pan? We regard it as a two-dimensional heat conduction problem. Discretization and Numerical analysis can emulate this progress using computer software. But Numerical analysis always involves dealing with a huge mass of analytic expressions, which makes the model building process hard and complex.

The method with an acceptable precision and easier to build is the Mount Carlo simulation method. For other assists, since the pattern of light radiation and the pattern of heat radiation are somehow similar, using the light simulation software DIALux to simulate the process of heat emission would be a reasonable choice.

All these models ignore the influence of heat convection, because in a small enclosure, the effect of convection is quite slight.

Finally we find the functional relationship between roundness (a parameter we defined) and uniformity coefficient.

Based on the analysis above, we can figure out a solution for the second question. When putting pans in an oven----which is rectangular in most cases----to bake things, it’s apparent that using round pans is not efficient enough with respect to oven-space using. So we try to decide the shape of pans put into an oven that have the best over-all properties over-all properties. We think the number of racks in an oven has nothing to do with heat transfer, so we study only one rack in our model.

We study the size of ovens used nowadays, and find that they almost have only two sizes: a large size $L_1 \times W_1$ and a small size $L_2 \times W_2$. Since the area of each pan must is fixed, so we then aim to help decide what kind of pans (only differ in shape) to be put in these two types of ovens to meet different conditions:

Condition1: maximum number of pans to be put into an oven;
Condition2: maximum evener distribution of heat for a pan; and
Condition3: optimize a combination of Need1 and Need2 where weights p and $(1-p)$ are assigned to illustrate how the results vary with different values of W/L and p;

We study four types of a pan: round-shaped, rectangle-shaped, square-shaped, and the Ultimate Brownie Pan we designed. After lying out and analytical calculation, we find out the solutions with regard to the above needs.
Section 1. Heat Distribution Problem----the models of heat distribution across pans

1.1 Assumptions

In order to find out the connection between heat distribution and the shape of a pan, we make the following assumptions:

1. **Pan-&-Oven Heat Source** The pans can be treated as a part of oven, and once pans are put into oven, it can get the same temperature of the oven and will never change (since ovens have automatic temperature regulator to maintain its temperature).

2. **Above-air media.** It is a kind of media in which the heat can dissipate.

3. **Imagined-air media.** This imagined-air media actually stands for the product we study. This imagined-air media has different temperatures in different parts and it will transfer internal heat.

4. **Two-dimensional.** We suppose the transference of heat in the imagined-air layer is a kind of two-dimensional problem, namely, the temperature and amount of heat in this layer are the same vertically.

1.2 The Models

1.2.1 Model 1: RADIATION MODEL

1.2.1.1 Build the model:

Since energy can conduct quite quickly in iron and steel, we define thermo dynamical parameters (including temperature) of Pan-&-Oven (imagine it as a complete combination of pan and oven) are the same. In order to analyze products’ temperature distribution, we study a layer of imagined-air. The imagined-air defined here is a layer of **Energetic sources** just above the pan without thickness. And **Energetic Particles** stand for particles that have a constant amount of energy $E_j$.

We divide imagined-air and the Pan-&-Oven layer (under and around the imagined-air) into N and M small units $IM_i$ ($1 \leq i \leq M$ ) and $PO_i$ (Energetic sources) $1 \leq i \leq M$.
In this ideal model, we take thermal radiation into consideration only; therefore, each IM (imagined-air) unit gets energy from PO (Pan-&-Oven) units.

The Stefan–Boltzmann law, also known as Stefan's law, shows that the total energy radiated per unit surface area of a black body across all wavelengths per unit time (also known as the black-body irradiance or emissive power), $j^*$, is directly proportional to the fourth power of the black body's thermodynamic temperature $T$:

$$j^* = \sigma \cdot T^4,$$

where the constant of proportionality $\sigma$, called the Stefan–Boltzmann constant or Stefan's constant and the value of the constant is:

$$\sigma = 5.670400 \times 10^{-8} \text{ (} J\text{s}^{-1}\text{m}^{-2}\text{K}^{-4}\text{)},$$

and $j^*$ only depends on $T$.

So if $T$ is given, we know that, the total energy point A radiates to point B $j(a, b)$ has a relationship with their inter distance ‘$r$’ only.

![Figure 1](image1.png) **Figure 1** the pattern of the PO units

![Figure 2](image2.png) **Figure 2** the radiance diagrammatic sketch of a unit

Apparently, in three-dimensional space, $j^*$ is distributed evenly on a spherical surface; and in this two-dimensional space problem, $j^*$ is distributed evenly on a ring. We then obtain this formula:
\[ j(a, b) \propto j^* \frac{1}{r^2} \]  \hspace{1cm} (1.2)

Define every PO units emit \( K \) Energetic Particles to any IM units in unit time, and the number \( K \) will not change since we suppose the energy PO units lost will immediately be replenished, making them a stable energy provider.

Additionally, the number of Energetic Particles (\( Num \)) that each unit contains reflects the amount of energy (\( E \)) it has, so we get:

\[ E = f(Num) = t \cdot Num, \]  \hspace{1cm} (1.3)

where \( t \) is a constant multiple.

And to test whether the particles can be absorbed, a parameter called the radiation rate is defined. If the radiation rate is higher than the probability threshold value, the particle will be absorbed; if lower, the particle will be bounced away.

We use Monte Carlo Method to simulate this Probability Statistical Process: \( K \) Energetic Particles transfer from unit \( PO_i \) to unit \( IM_j \), and from (1.2) the probability whether it can be absorbed by unit \( IM_j \) is:

\[ p = p_1 \frac{r^2}{r_0^2} \]  \hspace{1cm} (1.4)

where \( r \) stands for the distance between the emitting PO unit and the receiving IM unit, and \( r_0 \) represents the distance between the nearest emitting PO units and the receiving IM units.

If the PO unit is just on the same site with the IM unit or within a very small distance, we can be sure that the threshold value is large enough for all the particles to be absorbed by the IM unit.

Besides the PO unit mentioned above, the \( p_1 \) is the probability threshold value of the PO unit nearest to the IM unit, and the \( r_0 \) is the distance between the above two. In the practical calculation process, we define the \( p_1 \) value as 0.75. If the distance between a PO unit and a IM unit increase, the probability threshold value will relatively decrease.

We then sum up the total amount of Energetic Particles that \( IM_j \) gets from \( PO_i \) per unit time.

\[ Sum(IM_j, PO_i) \]  \hspace{1cm} (1.5)

Therefore, when calculating the number of Energetic Particles that \( IM_j \) get from the PO units, we use:

\[ SUM = \sum_{i=1}^{N} Sum(IM_j, PO_i) \]  \hspace{1cm} (1.6)

From (1.3), we can draw the conclusion that the overall energy that an IM unit gets from Pan-&-Oven is:

\[ E = f(SUM) = t \cdot SUM \]  \hspace{1cm} (1.7)

\[ E = f(SUM) = t \cdot \sum_{i=1}^{N} Sum(IM_j, PO_i) \]  \hspace{1cm} (1.8)

For each PO unit, we assume that on each direction, the unit generates 1000 Energetic Particles for the radiation process; to be more specific, each IM unit may receive 1000 Energetic Particles from the PO unit that is interfering with the IM unit.

Next step is we simulate whether the 1000 particles mentioned above will be
absorbed by the IM unit.

**Figure 3** the diagram for the simulation process, number of particles will be 1000.

The probability threshold value equals $p$ in (1.4)

The method is based on the probability and random sampling to simulate a process or to obtain a certain result.

Statistically speaking, the radiation rate is a certain number between 0 and 1. The random numbers generated is evenly distributed. The particle that satisfies the condition below will be absorbed, while the others will be bounced away.

*Random number between 0 and 1 $\leq p$(radiation rate) (1.9)*

We assume that the particle that is bounced away will leave the system, so the pure gain will only be the particle absorbed. The sum of the energetic particles received represents the total energy received.

The most common Monte Carlo simulation is the area calculation process. For example, if we have an irregular area inscribed in a square, we can calculate the area of the irregular parts by randomly throwing small stones into the square, and calculate the number of stones that falls in the irregular parts. If the number is N and M stones are totally thrown, we can say that the area of the irregular part is:

$$A = S_{square}\frac{N}{M}$$  \hspace{1cm} (1.10)

**1.2.1.2 Simulation process and the results**

The simple simulation process is shown below.

**Figure 4** the simulation process of the model 1

Take the example that we have 144 IM units and 188 PO units; we draw a contour map to show the Monte Carlo Method simulation result. The area of the pan will be 144, and the length and the width will be 12, making it a square. (See Figure 5 for detail)
On the other hand, the process of radiation is much similar to the radiation of the light in an enclosure.

The Luminous flux is in fact the radiance intensity of the sight visible to the human beings, and the value of this variable is not only based on the radiation intensity, but also related with the characteristics of the human eyes. Although it cannot fully match the radiance intensity in this problem, it can somehow reflex the trend of the varying intensity, which will be a good reference for us.

The Best software that can simulate the light radiation process is the DIALux. DIALux can build up a 3-D model for an enclosure, and calculate the radiation intensity on every unit area. Figure 6 shows the situation that when there are 188 PO units and 144 IM units. Each of the squares in the drawing represents a lamp, and the lamp in the figure represents a PO unit, so the radiation intensity on the surface just above the lamp is similar to the radiation intensity on the IM units. Figure 7 shows the simulation results, comparing with the result of our Monte Carlo Model.

Figure 5: The contour map of the heat distribution

Figure 6: The impression drawing of the light radiation process generated by DIALux.

And the simulation result is listed below.
Figure 7 picture (a) is the simulation result of the DIALux, numbers on the figure shows the radiation intensity on the point. Picture (b) is the simulation result of our program. There are $8 \times 18$ PO units in this simulation process.

To clearly demonstrate the heat distribution in various kinds of pans, we also simulate the other four kinds of pans, with the same area of 144 and different lengths and widths. The four other rectangles are $9 \times 16$, $6 \times 24$, $4 \times 36$. The simulation results are listed below. (See Figure 8)
1.2.2 Model 2: S-CONDUCTION MODEL

1.2.2.1 Build the model

The model above considers heat emission merely, but the heat radiation mostly takes effect in far-distance heat transmission. So in fact, the main heat transference will consequently be s-conduction.

In order to distinguish the concept of conduction we use here with the word ‘conduction’ we know in Physics, we form a word: s-conduction. In Physics, conduction happens when two contacted objects or two adjacent parts of an object have different temperature, and heat conducts from higher-temperature parts to lower parts. But s-conduction happens so long as two parts are adjacent.

Similarly, the pan and the oven are regarded as a whole, and once a pan is put into an oven, we assume Pan-&-Oven can attain the same temperature of the oven’s settled temperature $T_M$, but the imagined-air as well as the above air’s temperature still remains at an initial low temperature $T_0$. This assumptive condition is reasonable because solid medium can conduct quite quickly.

In this model, the number of Energy Particles is also used to give a visual representation of heat. And when analyzing imagined-air, we use cubic units (see Figure 9). Each unit has six surfaces and Energy Particles transmit through these surfaces.

Since there are three kinds of material that these surfaces adjacent to, s-conduction happens in three modes, and as a result, the loss and gain of Energy Particles consist of 3 parts:

Part1: Between imagined-air units and Pan-&-Oven units. (Red parts in Figure 9)
Part2: Between imagined-air units. (Yellow parts in Figure 9)
Part3: Between imagined-air units and the upper air units. (Blue parts in figure 9)

Guillermo Araya (2006) tells that the transient three-dimensional heat conduction equation:

$$\frac{1}{\alpha_T} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + \frac{q'''(x,y,z,t)}{k_T}$$

(1.11)

Where $k_T$ and $\alpha_T$ are the thermal conductivity and thermal diffusivity of the work piece respectively, $q'''(x,y,z,t)$ is the heat generation source term, and $\theta$ stands for
the temperature difference of two parts, i.e.
\[ \theta = T - T_0 \]  
(1.12)

In fact, \( \theta \) will affect the amount of energy transmitted, but in order to simplify the model, we assume that neighboring units can get the same temperature quickly, so:
\[ \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial y^2} = \frac{\partial^2 \theta}{\partial z^2} = 0 \]  
(1.13)

So the amount of Energy Particles transmitted through a surface just depends on the heat generation source term \( q''(x, y, z, t) \).

Whatever the material is, it emits and absorbs energy at one time. But we can simplify these changes because we know Pan-&-Oven units heat up an imagined-air unit, above-air units cool down it, and other imagined-air both heat it up and cool it down. For the surface between a Pan-&-Oven unit and an imagined-air unit in unit time, the number of Energy Particles a Pan-&-Oven unit wants to emit is \( N_1 (N_1 = 10000) \) (corresponds to Part 1);

For the surface between two imagined-air units in unit time, the number of Energy Particles each imagined-air unit wants to emit is \( N_2 (N_2 = 2000) \) (corresponds to Part 2);

As for the surface between an imagined-air unit and an above-air unit, the number of Energy Particles that an imagined-air unit wants to emit is \( N_3 (N_3 = 1000) \) (corresponds to Part 3);

\( N_1, N_2 \) and \( N_3 \) depend on heat generation sources’ temperature. And \( N_1 > N_2 > N_3 \) because Pan-&-Oven is hotter than imagined air, and imagined air is hotter than above air.

**Absorption probability**

When thinking of the probability that an Energy Particle is absorbed by an imagined-air unit, we think it depends on the neighboring units’ capacity of heat transmission.

We then define \( A_p \) as absorbing capability parameter, which is a percentage and stands for the ability an Energy Particle can successfully absorbed by the receiver. \( A_p \) of the above three parts are apparently different. And they are \( A_{p1}(A_{p1}: PO \text{ emit}, IM \text{ absorb}) \), \( A_{p2}(A_{p1}: IM \text{ emit}, IM \text{ absorb}) \) and \( A_{p3}(A_{p1}: IM \text{ emit}, air \text{ absorb}) \) respectively.

\( A_{p1}(A_{p1} = 0.5) \), \( A_{p2}(A_{p2} = 0.4) \) and \( A_{p3}(A_{p3} = 1) \).

**Emission probability**

For the same reason, we define \( B_p \) as emitting capability parameter. They are \( B_{p1}(the \text{ emitting ratio of PO to IM}) \), \( B_{p2}(the \text{ emitting ratio of IM to IM}) \) and \( B_{p3}(the \text{ emitting ratio of IM to air}) \) corresponding to Part 1, Part 2 and Part 3 respectively. In this problem, the Pan-&-Oven units have a fixed temperature, so the energy it lost will be replenished. Consequently, we can directly set \( B_{p1} = 1, B_{p2} = 1, B_{p3} = 0.3 \).
Example

To state it more clearly, we analyze IM₁ as an example here:

![Image of IM unit](image)

**Figure 10** One of the IM unit, here we use this unit as an example

IM₁ has six surfaces, S₁, S₂, S₃...S₆. And S₁, S₂, S₃ contact Pan-&-Oven directly; S₄ and S₅ contact IM₂ and IM₃ respectively; and S₆ contact the above-air.

Absorbing

For the particle receiving process, similar to the emission and acceptance process in model 1, N₁ particles emit from PO₁, PO₁₃ and PO₂₆ to IM₁ through surfaces S₁, S₂ and S₃. Since the absorbing capability parameter is Aₚ₁, Using Monte Carlo Method we can count the particle number that surface S₁, S₂, S₃ gets is respectively Accept₁(N₁), Accept₂(N₁), Accept₃(N₁).

Obviously, Acceptₚₒ(N₁) depends on N₁ and Aₚ₁.

N₂ particles emit from IM₂ and IM₃ to IM₁ through surfaces S₄ and S₅. We can also derive that the particle numbers that surface S₄ and S₅ gets are respectively Accept₄(N₂), Accept₅(N₂).

Likewise, Acceptᵢₘ(N₂) depends on N₂ and Aₚ₂.

Emitting

For the particle receiving process, N₂ particles want to emit from IM₁ to IM₂ and IM₃ through surfaces S₄ and S₅. And the particle numbers that surfaces S₄ and S₅ lost are respectively Lost₄(N₂), Lost₅(N₂).

As we can see, Lostᵢₘ(N₂) depends on N₂ and Bₚ₂.

N₃ particles emit from IM₁ to A₁₁ through surfaces S₆. And the particle numbers that surface S₆ lost actually is Lostᵦᵢr(N₂), while the Lostᵦᵢr(N₂) depends on N₃ and Bₚ₃.

The sum

Thus, we can thus derive the total amount a unit get is:

\[ \text{Obtain} = \sum_{i=1}^{6} \text{Accept}(N_i) - \sum_{i=1}^{6} \text{Lost}(N_i) \]  \hspace{1cm} (1.14)

Then using (1.3), we can calculate the amount of energy E₂ that this unit IM₁ gets or loses.

\[ E_2 = f(\text{Obtain}) = t \cdot \text{Obtain} \]  \hspace{1cm} (1.15)
1.2.2.2 Simulation process and the results

Simulating the process using Monte Carlo Method gives us the result as below:

![Simulation result](image11.png)

**Figure 11** the simulation result

1.2.2.3 Sensitivity analysis and modification.

From the above results, we can find out that the difference between the maximum part of the distribution and the minimum part is apparently too large, making the results different from the practical consequences. So, we engage the parameter modification process.

First, we think a rise in the value of \( N_2(N_2 = 5000) \) is necessary.
Second, we modify the value of \( A_{p2} \) to \( A_{p2} = 0.78 \).
Third, we reduce the value of \( B_{p2} \), \( B_{p2} = 0.4 \).

After modification, the simulation results are listed below:

![Simulation result after modification](image12.png)

**Figure 12** result after modification
1.2.3 Model 3: RADIATION & CONDUCTION MODEL

1.2.3.1 Build the model:

Since conduction, convection and thermal radiation are the three methods of heat transfer, and in this problem, all the media are actually solid, ignoring convection is reasonable. However, the other two ways of heat transfer can’t be neglected. So we know Model 1 and Model 2 have their limitations because Model 1 considers thermal radiation only and Model 2 only conduction.

Therefore, we build Model 3, which is actually a sound combination of Model 1 and Model 2. Model 3 considers influencing factors more comprehensively.

Heat can be superimposed, and in our models, energy is represented by Energetic Particles. So we believe that the terminal distribution of energy is a sum of energy in Model 1 and energy in Model 2.

That is:

$$E = E_1 + E_2$$  \hspace{1cm} (1.16)

However, when using Monte Carlo Method to calculate the total number of Energetic Particles, two problems appear:

Pro 1: The importance of conduction and thermal radiation is different.

Pro 2: What if the property relation between simulating numbers used in Model 1 and Model 2 are not reasonable.

In order to deal with these two problems, we can indicate a weight $\alpha$, so:

$$TOTAL = \alpha SUM + (1 - \alpha) Obtain$$  \hspace{1cm} (1.17)

1.2.3.2 Simulation process and the results

We simulated the consequence when $\alpha = 0.2, 0.4, 0.6, 0.8$, based on the $12 \times 12$ square. (for results, see Figure 13)
We think the result of $\alpha = 0.6$ is better than the other ones, so we check the result when $\alpha = 0.57, 0.65, 0.67, 0.7$.

From the above figures we can infer that, when $\alpha = 0.67$, the heat distribution will mostly satisfy the practical situation, so we choose $\alpha = 0.67$.

A shape’s roundness refers to the similarity between the shape and a real-life circle. The girth of a circle is proportional to its diameter, the ratio is $\pi = 3.14$. For other shapes, we can use the specific value between its girth and the longest distance between two points on the edge of the shape. To compare this value with $\pi$, we define the ratio between the roundness and $\pi$ as the ratio of roundness. The line mentioned above should specifically pass through the center of the shape. Typically, if the shape is a regular polygon, the longest distance will be the longest diagonal line. Take the picture below as an example. To be convenient, we define the roundness as $ro$, while the ratio of roundness as $RO$. 
For the shapes we use in this section, their roundness is just as follow:

<table>
<thead>
<tr>
<th>The shape</th>
<th>12 × 12</th>
<th>9 × 16</th>
<th>8 × 18</th>
<th>6 × 24</th>
<th>4 × 36</th>
</tr>
</thead>
<tbody>
<tr>
<td>The roundness ((r_o))</td>
<td>2.82</td>
<td>2.72</td>
<td>2.64</td>
<td>2.43</td>
<td>2.21</td>
</tr>
<tr>
<td>The ratio of roundness ((RO = \frac{r_o}{\pi}))</td>
<td>0.898</td>
<td>0.866</td>
<td>0.840</td>
<td>0.774</td>
<td>0.704</td>
</tr>
</tbody>
</table>

Based on the data of heat distribution, we analyze the data on the outer edge of the data. The data reflects the heat distribution of the outer edge. Now we will define a parameter that can reflect the rate of unevenness of the distribution: Coefficient of variation. The parameter is the ratio between the Standard Deviation and the Mean Value.

\[ CV = \frac{\sigma}{\mu} \]  \hspace{1cm} (1.18)

And the results of the Coefficient of variation are as follows:

<table>
<thead>
<tr>
<th>The shape</th>
<th>12 × 12</th>
<th>9 × 16</th>
<th>8 × 18</th>
<th>6 × 24</th>
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</tr>
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<td>2.72</td>
<td>2.64</td>
<td>2.43</td>
<td>2.21</td>
</tr>
<tr>
<td>The coefficient of variation</td>
<td>0.0209</td>
<td>0.0247</td>
<td>0.0283</td>
<td>0.0298</td>
<td>0.0322</td>
</tr>
</tbody>
</table>

We do the linear fitting. The x-axis stands for the roundness, while the y-axis stands for the coefficient of variation. The result is shown in Figure 16.

Figure 15 the longest distance on a shape for a rectangle and a regular hexagon

Figure 16 the image of the linear curve fitting
While the result of the curve fitting tool is just like this:

\[ CV = 0.0705 - 0.0531\text{RO} \]  

(1.19)

And the goodness of fit is as follows:

- SSE: 1.068e-05
- R-square: 0.8647
- Adjusted R-square: 0.8196
- RMSE: 0.001886

1.3 Conclusions

In Model 1, the center of the pan has more power, while there is less heat on the edges. The distribution of heat across the outer edge of a pan depends on the shape. The further an edge is from the center, the more energy it can get. So, the corner is hotter than the adjacent side, which agrees with fact.

In Model 2, the edge of a pan has more power, while there is less heat on the center. As for the distribution of heat across the outer edge, it turns out to have the same qualitative law as Model 1.

From Model 3, we derive that the coefficient of variation of a circle is 0.0174, which is the smallest, and has a minor difference between the theoretical values of 0. As the ratio of roundness increases, the value of the coefficient of variation decrease. And the formula is

\[ CV = 0.0705 - 0.0531\text{RO} \]

1.4 Strengths and Weaknesses & Future Work

Strengths for Model 1&Model 2:

1. We use Monte Carlo Method instead of Analytic Method. So these two models can well describe the characteristics when tackling with mass particles.
2. It is reasonable and can relatively well reflect the true process of heat transfer.

Strengths for Model 3:

1. It considers all the influencing factors we analyzed in Model 1 and Model 2, so it is more comprehensive, more reasonable and more scientific. It reflects reality better.
2. It concludes a formula, and it makes the evaluating of heat distribution more easily.

Weaknesses for Model 1&Model 2:

1. Both of them neglected heat convection, which obviously infects the final heat an imagined-air can get.
2. They neglected heat conduction and thermal radiation respectively. And this makes the analysis one-sided.
Weaknesses for Model3:

1. It also neglected heat convection, so it is not very precise to some extent.
2. Likewise, in this model, the amount of energy a unit emits just depends on the material of it, and it doesn’t take the influence of interaction into account.

Future work:

Our future work will focus on refining the model to be more scientific and more believable; besides, some factors which are neglected in these models can be further studied.

For instance, we will find out the true experimental data of temperature and then modify our model; we will suppose that the energy a unit emits is time-variant; we will also find out how the materials of pans or ovens influence the heat transfer process.

Section 2. The Pan-Shape-Choosing Problem

2.1 Assumptions

1. In this problem, we still use some conclusions that we draw from the upper models, so the above-mentioned common assumptions we make are still valid here.

2. There are evenly spaced two racks in the oven initially, but we think that these two racks will not affect each other. So, to make the problem more easily, we study only one rack in the oven and we think two or three or even four racks will all turn out to be the same.

2.2 Preparation work

After searching the size of ovens in the Internet, we find that:

<table>
<thead>
<tr>
<th>The type of the Oven</th>
<th>The shape of the oven (W × L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siemens HB84H500W</td>
<td>594mm × 570mm</td>
</tr>
<tr>
<td>Midea EAC56AQ-ERS</td>
<td>590mm × 600mm</td>
</tr>
<tr>
<td>Siemens HB23AB520W</td>
<td>560mm × 550mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The type of the Oven</th>
<th>The shape of the oven (W × L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACA ATO-MR23A</td>
<td>310mm × 315mm</td>
</tr>
<tr>
<td>Midea MG25AF-PRR</td>
<td>337.5mm × 323.5mm</td>
</tr>
</tbody>
</table>

So it’s reasonable to define two ovens: Oven 1: a large size, \( L1 \times W1 = 59cm \times \)
58cm; and Oven 2: a small size $L2 \times W2 = 32cm \times 31cm$.
And the sizes of pans we find are as below:

<table>
<thead>
<tr>
<th>Pan</th>
<th>Shape</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pan(big)</td>
<td>320mm x 210mm</td>
<td></td>
</tr>
<tr>
<td>Pan(small)</td>
<td>240mm x 130mm</td>
<td></td>
</tr>
</tbody>
</table>

We want to study the smaller ones, then we define all the pans we use have the same area $A$, and:

$$A = 24cm \times 13cm = 312cm^2$$

(2.1)

So we study three kinds of pans here first:
- Pan 1: oblong pan, $L \times W = 24cm \times 13cm$;
- Pan 2: square pan, $L = 17.66cm$; and
- Pan 3: round pan, $R = 19.92cm$;

2.3 Question 1: What is the maximum number of pans (N) that can fit in the oven?

The method of obtaining the layout contains two steps: first, we roughly determined the division of the area, and we use a Lingo program to calculate the precise position of the pan.

In Oven 1, the best layouts of pans are respectively:

![Figure 17](image1.png)

So the numbers of pans (N) that can fit in it are:

$$N(Pan\ 1) = 8;$$
$$N(Pan\ 2) = 9;$$
$$N(Pan\ 3) = 7$$

(2.2)

In Oven 2, the best layouts of pans are respectively:

![Figure 18](image2.png)

So the numbers of pans (N) that can fit in it are:

$$N(Pan\ 1) = 2;$$
$$N(Pan\ 2) = 1;$$
\[ N(Pan\ 3) = 1 \] (2.3)

After the calculation above, we can infer that:
1. For large sized pans (59cm × 58cm) many people use, using square pans can make the number of pans in the oven the largest; and
2. For small sized pans (32cm × 31cm) many people use, using oblong pans can make the number of pans in the oven the largest.

2.4 Question 2: What is the maximum even distribution of heat (H) for the pan?

In this problem, we can surely simply use the formula concluded from Model 3, but since a product is a whole, we think that relative extreme difference \((R_{ed})\) works better to reflect a standard of the evenness of the heat distribution, i.e. \(R_{ed}\) reflects H, and:

\[
R_{ed} = \frac{\text{TOTAL}_{\text{max}} - \text{TOTAL}_{\text{min}}}{\text{TOTAL}_{\text{ave}}} \quad (2.4)
\]

where \(\text{TOTAL}_{\text{max}}\), \(\text{TOTAL}_{\text{min}}\) and \(\text{TOTAL}_{\text{ave}}\) are the maximum number, the minimum number and the average number that imagined-air units get eventually in model 3.

Although \(R_{ed}\) and \(CV\) from Model 3 both represent the evenness of distribution, the \(R_{ed}\) has an effect on the whole area, and represents the relative extreme difference; while \(CV\) only focuses on the outer edge of the pan, and represents the coefficient of variance.

Where \(\text{TOTAL}_{\text{max}}\), \(\text{TOTAL}_{\text{min}}\) and \(\text{TOTAL}_{\text{ave}}\) are the maximum number, the minimum number and the average number that imagined-air units get eventually in model 3.

From the statistics in Model 3, we can have the below table:

<table>
<thead>
<tr>
<th>The shape</th>
<th>12 × 12</th>
<th>9 × 16</th>
<th>8 × 18</th>
<th>6 × 24</th>
<th>4 × 36</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ratio of roundness</td>
<td>0.898</td>
<td>0.866</td>
<td>0.840</td>
<td>0.774</td>
<td>0.704</td>
</tr>
<tr>
<td>Relative extreme difference</td>
<td>0.818</td>
<td>0.806</td>
<td>0.798</td>
<td>0.789</td>
<td>0.780</td>
</tr>
</tbody>
</table>

The curve fitting result will be:

\[
R_{ed} = -0.182RO + 0.351 \quad (2.5)
\]

While the image of the fitted curve is:
Goodness of fit:
SSE: 4.968e-05
R-square: 0.9436
Adjusted R-square: 0.9248
RMSE: 0.004069

We run the simulation procedures and using statistical knowledge to derive the relative extreme difference ($R_{ed}$) of the three types of pans as follow:

<table>
<thead>
<tr>
<th>Shape of the pan</th>
<th>RO</th>
<th>$R_{ed}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24 \text{cm} \times 13 \text{cm}$</td>
<td>0.863</td>
<td>0.194</td>
</tr>
<tr>
<td>$17.66 \text{cm} \times 17.66 \text{cm}$</td>
<td>0.900</td>
<td>0.187</td>
</tr>
<tr>
<td>$\text{Circle, radius } = 19.92 \text{cm}$</td>
<td>1</td>
<td>0.169</td>
</tr>
</tbody>
</table>

From the above model, we can know that whatever the size of pans people use, round pans can produce the most even products.

2.5 Question 3: If the two questions above are both asked to take into consideration, what will be the best proposal of pan-choosing?

We infer that Pan1, Pan2 and Pan3 may all fail to work out efficiently, so we design a new kind of pan, and name it Pan4.

The four kinds of pans are shown below:

![Figure 21](image) the pans discussed in this section. From left to right, the pan is pan1, pan2, pan3, pan4
In the assumption part, we’ve figured out the shape of the first three pans. In order to make the problem-solving easier, we define a number $R_n$ without dimension and with the following substitutions. $R_n$ is the ratio between area covered by the pans and the whole area of the oven, which in fact represents the efficiency of the area.

$$R_n = \frac{N}{N_{\text{max}}} = \frac{AN}{S}$$

(2.6)

where $N_{\text{max}} = \frac{S}{A}$, S is the whole area of the oven, and A is the area of each pan.

To define the evenness of the heat distribution, we use $R_h = 1 - R_{ed}$, where $R_{ed}$ is just the degree of unevenness mentioned above.

And we use $m$ to represent the preference of the pan, so we can get:

$$m = p \times R_n + (1 - p) \times R_h$$

(2.7)

where $p$ is a weight here which actually indicates the importance of space-using efficiency. (If a user thinks the amount that the oven can cook is more important, $p$ is larger; on the other hand, if the user wants to have better-cooked products, $p$ is smaller.)

In order to consider two different factors at the same time, it is important to introduce non-dimensional coefficients. For the newly designed pan, the best layout will be:

![Figure 22 the layout of the newly designed pan](image)

And with the statistic from Question 1, we can calculate the data in the table below (using oven with a shape of 59cm × 58cm):

<table>
<thead>
<tr>
<th></th>
<th>Pan1</th>
<th>Pan2</th>
<th>Pan3</th>
<th>Pan4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_n$</td>
<td>0.729</td>
<td>0.821</td>
<td>0.638</td>
<td>0.821</td>
</tr>
<tr>
<td>$R_h$</td>
<td>0.806</td>
<td>0.813</td>
<td>0.831</td>
<td>0.826</td>
</tr>
</tbody>
</table>

Table 8 the $R_n$ and $R_h$ for the pans

Take the above data into consideration; we can have the upcoming results:

1. If $p \leq 0.027$, using pan3 is the best solution;
2. if $p > 0.027$, using pan4 is the best solution.

If someone needs the evenness of the food cooked in the pan to be extraordinary good, he should consider pan3; if people do not persist on the evenness or want more food to be cooked at the same time, pan4 would be the best choice.

Basically, the oven in practice often has the property that $\frac{W}{L} \approx 1$, and the size of the oven is often 32cm × 31cm or 59cm × 58cm, and though other dimensions may not precisely suite the above data, the difference would be a little. For the oven with a size of 32cm × 31cm, there will only be room for one or two pans, so considering the efficiency of the area is somehow meaningless; choosing the pan with the best relative
extreme difference would be a better choice.

For other sizes, we’ve designed a kind of area-variable pan that can not only suit the need of increasing the pan used at the same time, but also take the heat distribution into consideration. The flexible pan we designed is shown in Figure 23. From the pan41 on the left to the pan44 on the right, we can find out that, $R_n$ is decreasing, while the $R_h$ generally increase. The detailed data is listed in Figure 24.

Figure 23 the diagrammatic sketch of the flexible pan, from the left to the right is pan41, pan42, pan43 and pan44.

<table>
<thead>
<tr>
<th>$R_n$</th>
<th>$R_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.813</td>
</tr>
<tr>
<td>0.900</td>
<td>0.822</td>
</tr>
<tr>
<td>0.841</td>
<td>0.825</td>
</tr>
<tr>
<td>0.759</td>
<td>0.826</td>
</tr>
</tbody>
</table>

Figure 24 the $R_n$ and $R_h$ of the flexible pan above, from the left to the right is the data for pan41, pan42, pan43 and pan44.

From the data analysis we can see that, if we sacrifice the efficiency of area, the gain speed of the evenness would become slower, thus we do not suggest applying the pan44, and the pan42 would be a better choice.

And if we want the flexible pan put into practice, we will design the pan like this: (Figure 25)
As shown in the picture, the flexible pan in real life has four plates, and owners can adjust the position of the plate freely. When the plate is placed near the corner, the pan will have the same effect as a square-shaped pan; if there is a distance between the plate and the corner, the pan will have a similar effect as a round pan. Thus, the plate will be flexible and can apply to various situations easily.

2.6 Strengths and Weaknesses & Future Work

Strengths:

1. Our model is practical, and it turns out to be very good when tackling with real-life choosing problem.
2. It has considered many factors, and it can meet the need of different people, that is to say, it can be widely used and it seems universal.

Weaknesses:

1. We over simplify the model because we suppose two racks in an oven performs just like one rack. However, in fact, pans in upper racks will get less energy and thus, products differs as the rack it is in differs.
2. We suppose every pan in the same rack get the same energy; while actually, the oven can’t heat pans up absolutely evenly.
Future Work:

Our future work will aim to solve the weaknesses we find in the model, namely, we will try to consider more factors.

2.7 Conclusions

For the above three questions, we can draw the conclusion for each of them.

For question 1, the large sized pans (59cm × 58cm) many people use, using square pans can make the number of pans in the oven the largest; and for small sized pans (32cm × 31cm) many people use, using oblong pans can make the number of pans in the oven the largest.

For question 2, the relationship between the roundness and the relative extreme difference:

\[ R_{ed} = -0.182RO + 0.351 \]

At the same time, we can know that round pans can produce the most even products.

For question 3, we’ve found that no matter a customer wants to maximum the product or the quality of the food, pan4, which is the square with its four corners cut away, will be a good choice. Under 97.3% of the circumstances, pan4 will suit people’s need.

Synthesizing the results in the above three problems, we’ve designed a kind of area-variable pan that can increase the number of pan being cooked and increase the evenness of the heat distribution on each pan as well.

References

**Recommendations:** the advertising sheet for the new Brownie Gourmet Magazine

Dear sir or madam,
Thank you very much for spending time reading my letter.
As you can see, we all face a practical but troublesome situation: how to choose a better pan. Most sold pans are either square or round. For the round ones, we will all be troubled by the fact that there is often not so much room for the pans to stay. In order to put more food, we need a rectangular pan, but the heat is often concentrated in its corners, so food may be overcooked.
This kind of “You cannot sell the cow and drink the milk” issue is pretty common in our daily life, and solution may always be nowhere. We cannot say we can handle the case, but using the pan designed by us, we can somehow make the case less troublesome.
We call our design “Ultimate Brownie Pan”. Other than the usual pans in the sea of life every day, our pan provides you with the high area-availability and high heat-efficiency at the same time. The picture below will give you a vivid description of the super power of our pans.

![Image of the Ultimate Brownie Pan](image1)

Thanks to the flexible plate, the shape of the pan can change in a limited but flexible scale. When the plate is set at the position near the corner, the heat concentration will decrease, and the area still remains to be pretty large; and if you make the plate stay away from the corner, the evenness of your food will increase, and you will find the effect would be much similar to our round pans.
If you focus on better taste, the flexible pan will give you the best quality and most amount; if you want to use the pan for your food business, the flexible pan will give you as much quality as the amount. The super power of our pan will make the cooking process easier and more enjoyable, and may be days later, we can all see the super pan waiting for your approach on the shelves of our super market.
Yours sincerely,
A dedicated designer